

d. (5 puntos) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 8x + 9} - \sqrt{x^2 - 8x + 9})$

$$\begin{aligned}
 & \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 8x + 9} - \sqrt{x^2 - 8x + 9}) \\
 = & \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 8x + 9} - \sqrt{x^2 - 8x + 9}) \frac{\sqrt{x^2 + 8x + 9} + \sqrt{x^2 - 8x + 9}}{\sqrt{x^2 + 8x + 9} + \sqrt{x^2 - 8x + 9}} \\
 = & \lim_{x \rightarrow +\infty} \frac{(x^2 + 8x + 9) - (x^2 - 8x + 9)}{\sqrt{x^2 + 8x + 9} + \sqrt{x^2 - 8x + 9}} \\
 = & \lim_{x \rightarrow +\infty} \frac{16x}{\sqrt{x^2 + 8x + 9} + \sqrt{x^2 - 8x + 9}} \\
 = & \lim_{x \rightarrow +\infty} \frac{\frac{16x}{x}}{\frac{\sqrt{x^2 + 8x + 9} + \sqrt{x^2 - 8x + 9}}{x}} \\
 = & \lim_{x \rightarrow +\infty} \frac{\frac{16x}{x}}{\frac{\sqrt{x^2 + 8x + 9}}{x} + \frac{\sqrt{x^2 - 8x + 9}}{x}} \\
 = & \lim_{x \rightarrow +\infty} \frac{\frac{16x}{x}}{\sqrt{\frac{x^2 + 8x + 9}{x^2}} + \sqrt{\frac{x^2 - 8x + 9}{x^2}}} \\
 = & \lim_{x \rightarrow +\infty} \frac{16}{\sqrt{1 + \frac{8}{x} + \frac{9}{x^2}} + \sqrt{1 - \frac{8}{x} + \frac{9}{x^2}}}
 \end{aligned}$$

Por lo tanto:

$$\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 8x + 9} - \sqrt{x^2 - 8x + 9}) = 8$$