

3. (20%) Sean  $V = P_2$  y  $B_1 = \{x^2 + 1, x^2 - x, x - 3\}$  y

$B_2 = \{x^2 - 2x + 2, x - 3, x^2 - 1\}$  bases de  $P_2$ . Determine

a. La matriz de cambio de base de  $B_2$  a  $B_1$

b. El núcleo y la imagen (recorrido) de la matriz obtenida en (a) = ATRÁS

$$Op_2 \rightarrow p_1 = [x^2 - 2x + 2]_{p_1}, [x - 3]_{p_1}, [x^2 - 1]_{p_1}$$

$$x^2 - 2x + 2 = \alpha_1(x^2 + 1) + \alpha_2(x^2 - x) + \alpha_3(x - 3)$$

$$x^2 - 2x + 2 = \alpha_1 x^2 + \alpha_1 + \alpha_2 x^2 - \alpha_2 x + \alpha_3 x - 3\alpha_3$$

$$x^2 - 2x + 2 = x^2(\alpha_1 + \alpha_2) + (-\alpha_2 + \alpha_3)x + (\alpha_1 - 3\alpha_3)$$

$$\begin{cases} \alpha_1 + \alpha_2 = 1 \\ -\alpha_2 + \alpha_3 = -2 \\ \alpha_1 - 3\alpha_3 = 2 \end{cases}$$

$$\alpha_1 + \alpha_2 = 1$$

$$-\alpha_2 + \alpha_3 = -2$$

$$(3) \times \alpha_1 + \alpha_2 = 1 \quad 3\alpha_1 + 3\alpha_2 = 3$$

$$\alpha_1 - 3\alpha_3 = 2 \quad \alpha_1 - 3\alpha_3 = 2$$

$$-\frac{1}{4} + \alpha_2 = 1$$

$$\alpha_2 = 1 + \frac{1}{4} = \frac{5}{4}$$

$$\alpha_2 = \frac{5}{4}$$

$$-\frac{1}{4} - 3\alpha_3 = 2$$

$$-3\alpha_3 = 2 + \frac{1}{4}$$

$$-3\alpha_3 = \frac{9}{4}$$

$$-\alpha_3 = \frac{3}{4}$$

$$\alpha_1 = -\frac{1}{4}$$

$$x^2 - 1 = (\alpha_1 + \alpha_2)x^2 + (-\alpha_2 + \alpha_3)x + (\alpha_1 - 3\alpha_3)$$

$$\alpha_1 + \alpha_2 = 0$$

$$-\alpha_2 + \alpha_3 = 1$$

$$\alpha_1 - 3\alpha_3 = 2$$

$$\alpha_1 + \alpha_2 = 0$$

$$-\alpha_2 + \alpha_3 = 1$$

$$(3) \alpha_1 + \alpha_2 = 0 \quad 3\alpha_1 + 3\alpha_2 = 0$$

$$\alpha_1 - 3\alpha_3 = 2 \quad \alpha_1 - 3\alpha_3 = 2$$

$$4\alpha_1 = 2$$

$$\alpha_1 = \frac{1}{2}$$

$$\alpha_1 + \alpha_2 = 0$$

$$\alpha_2 = -\frac{1}{2}$$

$$\alpha_3 = \frac{3}{4}$$

Cambio =

$$\begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{2} \\ \frac{5}{4} & 0 & \frac{3}{4} \\ -\frac{1}{4} & 0 & \frac{3}{4} \end{pmatrix}$$

$$x^2 - 1 = (\alpha_1 + \alpha_2)x^2 + (-\alpha_2 + \alpha_3)x + (\alpha_1 - 3\alpha_3)$$

$$\alpha_1 + \alpha_2 = 0$$

$$-\alpha_2 + \alpha_3 = 1$$

$$\alpha_1 - 3\alpha_3 = 2$$

$$\alpha_1 + \alpha_2 = 0$$

$$-\alpha_2 + \alpha_3 = 1$$

$$(3) \alpha_1 + \alpha_2 = 0$$

$$3\alpha_1 - 3\alpha_3 = 2$$

$$3\alpha_1 - 3\alpha_3 = 2$$

$$\alpha_1 - \alpha_3 = \frac{2}{3}$$

$$3\alpha_1 = 2 + 3\alpha_3$$

$$\frac{1}{2} + \alpha_2 = 2$$

$$\alpha_2 = 2 - \frac{1}{2} = \frac{3}{2}$$