

4. (30%) Sea el espacio vectorial  $(V, \oplus, \odot)$  donde:

$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} / x, y \in \mathbb{R}, z \in \mathbb{R}^+ \right\}$  con las operaciones:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \oplus \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 + 2 \\ y_1 + y_2 \\ z_1 z_2 \end{pmatrix} \quad \alpha \odot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha x + 2\alpha - 2 \\ \alpha y \\ z^{\alpha-1} \end{pmatrix}$$

$$\begin{aligned} 3x_1 + 4x_2 &= 0 \\ -3\alpha_1 - 6\alpha_2 &= 0 \\ -2\alpha_2 &= 0 \\ \alpha_2 &= 0 \\ \alpha_1 &= 0 \end{aligned}$$

Determine:

a. El neutro de  $V$  y el opuesto de  $v \in V$

b. Si  $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  es combinación lineal de  $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$  y  $\begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$

no es comb. lineal xq!

$$\begin{aligned} 3x_1 + 4x_2 &= 0 \\ -\alpha_1 - 2\alpha_2 &= 0 \\ \alpha_1 + 2\alpha_2 &= 0 \\ \alpha_1 &= 0 \\ \alpha_2 &= 0 \\ \alpha_1 = \alpha_2 = 0 & \text{ comb. lineal} \end{aligned}$$

$$x \odot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x-2-2 \\ 0 \\ 1 \end{pmatrix}$$

$$x \odot (-1) \odot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x-2-2 \\ -y \\ z^{-1} \end{pmatrix} = \begin{pmatrix} -x-4 \\ -y \\ z^{-1} \end{pmatrix} = \begin{pmatrix} -x-4 \\ -y \\ 1/2 \end{pmatrix}$$

$$CL = \alpha_1 \odot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \oplus \alpha_2 \odot \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} \alpha_1 + 2\alpha_2 - 2 \\ -\alpha_1 \\ 2^{\alpha_1} \end{pmatrix} \oplus \begin{pmatrix} 2\alpha_2 + 2\alpha_2 - 2 \\ -2\alpha_2 \\ 4^{\alpha_2} \end{pmatrix} = \begin{pmatrix} 3\alpha_2 - 2 \\ -\alpha_1 \\ 2^{\alpha_1} \end{pmatrix} \oplus \begin{pmatrix} 4\alpha_2 - 2 \\ -2\alpha_2 \\ 4^{\alpha_2} \end{pmatrix}$$

$$\begin{pmatrix} 3\alpha_1 - 2 + 4\alpha_2 - 2 \\ -\alpha_1 - 2\alpha_2 \\ 2^{\alpha_1} \cdot 4^{\alpha_2} \end{pmatrix} = \begin{pmatrix} 3\alpha_1 + 4\alpha_2 - 2 \\ -\alpha_1 - 2\alpha_2 \\ 2^{\alpha_1 + 2\alpha_2} \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \begin{cases} 3x_1 + 4x_2 - 2 = -2 \\ -x_1 - 2x_2 = 0 \\ 2^{\alpha_1 + 2\alpha_2} = 2^0 = 1 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & 1/2 & 0 \\ 3/4 & 0 & 1/2 & 0 \\ 1/4 & 0 & 1/2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 & 0 \\ 5/4 & 0 & 1/2 & 0 \\ 1/4 & 0 & 1/2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 - 2y_2 &= 0 & x_1 &= 2y_2 \\ x_3 &= 0 & x_3 &= 0 \\ -x_3 &= 0 & x_3 &= 0 \end{aligned}$$

dim null = 2

$y_2 = 0$   
 $x_1 = 0$   
 $x_3 = 0$   
 $P(A) = 3P$

$$\begin{pmatrix} 1 & 0 & -2 & -4y_1 \\ 0 & 0 & 3 & y_2 = 5(-4y_1) \\ 0 & 0 & -1 & y_3 = \frac{3}{4}(4y_1) \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 & -4y_1 \\ 0 & 0 & 3 & y_2 = 5y_1 \\ 0 & 0 & -1 & y_3 = 3y_1 \end{pmatrix}$$