

$$b_n = \frac{2}{\pi} \left[-\frac{2}{n^3} (-1)^n + \frac{2}{n^3} \right]$$

$$b_n = \frac{4}{\pi n^3} \left[1 - (-1)^n \right]$$

$$b_n = \frac{4}{\pi n^3} \left[1 + (-1)^{n+1} \right]$$

$$\Rightarrow f(x) = \sum_{n=1}^{\infty} \frac{4}{\pi n^3} \left(1 + (-1)^{n+1} \right) \sin(nx)$$

$$\begin{array}{l} x=0 \Rightarrow f(x)=0 \\ \Rightarrow 0=0 \end{array} \quad \left| \quad \begin{array}{l} x=\pi \Rightarrow f(x)=0 \\ 0=0 \end{array} \right.$$

$$\Rightarrow x = \frac{\pi}{2} \Rightarrow f(x) = \frac{\pi^2}{4}$$

$$\Rightarrow \frac{\pi^2}{4} = \sum_{n=1}^{\infty} \frac{4}{\pi n^3} \left(1 + (-1)^{n+1} \right) \sin\left(\frac{n\pi}{2}\right)$$

$$\frac{\pi^2}{4} = \frac{8}{\pi} + \frac{8}{\pi(3)^3}(-1) + \frac{8}{\pi(5)^3} + \frac{8}{\pi(7)^3}(-1) + \frac{8}{\pi(9)^3} + \frac{8(-1)}{\pi(11)^3} + \dots$$

$$\frac{\pi^2}{4} = \frac{8}{\pi} - \frac{8}{\pi(3)^3} + \frac{8}{\pi(5)^3} - \frac{8}{\pi(7)^3} + \frac{8}{\pi(9)^3} - \frac{8}{\pi(11)^3} + \dots$$

$$\frac{\pi^2}{4} = \frac{8}{\pi} \left(1 - \frac{1}{(3)^3} + \frac{1}{(5)^3} - \frac{1}{(7)^3} + \frac{1}{(9)^3} - \frac{1}{(11)^3} + \dots \right)$$

$$\frac{\pi^2}{4} = \frac{8}{\pi} \left(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3} \right)$$

$$b) \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3} = \frac{\pi^3}{32}$$