$$\Rightarrow \sum_{n=0}^{\infty} a_n(n+r)(n+r-1)x^{n+r-1} - \sum_{n=0}^{\infty} a_n(n+r)x^{n+r-1} - \sum_{n=1}^{\infty} a_{n-1}(n+r-1)x^{n+r-1} + \sum_{n=1}^{\infty} a_{n-1}x^{n+r-1} = 0$$

$$\underbrace{a_0(r)(r-1)x^{r-1} - a_0(r)x^{r-1}} + \sum_{n=1}^{\infty}\left[a_n(n+r)(n+r-1) - a_n(n+r) - a_{n-1}(n+r-1) + a_{n-1}\right]x^{n+r-1} = 0$$

$$\Rightarrow \begin{array}{c} r^2 - r - r = 0 \\ r(r-2) = 0 \\ r_1 = 0,\ r_2 = 2 \end{array} \quad \Bigg| \quad a_n = \frac{a_{n-1}(n+r-1) - a_{n-1}}{(n+r)(n+r-1) - (n+r)}; \ \forall n \geq 1$$

$$\Rightarrow a_n = \frac{a_{n-1}\cancel{(n+r-2)}}{(n+r)\cancel{(n+r-2)}}; \ \forall n \geq 1$$

$$a_n = \frac{a_{n-1}}{n+r}; \quad \forall n \geq 1$$

$$a_n(r_1) = \frac{a_{n-1}}{n}; \ \forall n \geq 1$$

$n=1$

$$a_1 = a_0$$

$n=2$

$$a_2 = \frac{a_1}{2} = \frac{a_0}{2}$$

$n=3$

$$a_3 = \frac{a_2}{3} = \frac{a_0}{(2)(3)}$$

$n=4$

$$a_4 = \frac{a_3}{4} = \frac{a_0}{(2)(3)(4)}$$

$$y_1 = \sum_{n=0}^{\infty} a_n x^{n+0}$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$= a_0 + a_0 x + \frac{a_0}{2!}x^2 + \frac{a_0}{3!}x^3 + \frac{a_0}{4!}x^4 + \cdots$$

$$= a_0\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$$

$$y_1 = a_0 \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow y_1 = e^x$$

$$y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{(y_1)^2}dx = e^x \int \frac{e^{-\int -\frac{(1+x)}{x}dx}}{e^{2x}}dx$$

$$\Rightarrow y_2 = e^x \int e^{\ln|x| + x - 2x} dx = e^x \int e^{-x} \cdot x\, dx$$

$$\Rightarrow y_2 = e^x\left[-xe^{-x} - e^{-x}\right]$$

$$\Rightarrow y_2 = -x - 1 \ \vee \ y_2 = x + 1$$

$$\Rightarrow \underbrace{y(x) = c_1 e^x + c_2(x+1)}$$