$$\mathcal{L}^{-1}\left[\frac{d}{ds}F(s)\right] = \mathcal{L}^{-1}\left[\frac{1}{s-1} - \frac{2(s+1)}{(s+1)^2+4}\right] = -tf(t)$$

$$\Rightarrow \quad -tf(t) = e^{t} - 2e^{-t}\cos 2t$$

$$\Rightarrow \quad f(t) = \frac{2e^{-t}\cos 2t}{t} - \frac{e^{t}}{t}$$

---

c) $$y' + \int_0^t (t-u)y(u)\,du = t \; ; \; y(0) = 0$$

$$\mathcal{L}[y'] + \mathcal{L}\left[\int_0^t (t-u)y(u)\,du\right] = \mathcal{L}[t]$$

$$[sY(s) - y(0)] + \frac{1}{s^2}Y(s) = \frac{1}{s^2}$$

$$\Rightarrow sY(s) + \frac{Y(s)}{s^2} = \frac{1}{s^2} \Rightarrow s^3Y(s) + Y(s) = 1$$

$$\Rightarrow (s^3+1)Y(s) = 1 \Rightarrow Y(s) = \frac{1}{s^3+1}$$

$$Y(s) = \frac{1}{(s+1)(s^2-s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-s+1}$$

$$\Rightarrow 1 = A(s^2-s+1) + B(s^2+s) + C(s+1)$$

$A+B=0 \Rightarrow \boxed{A=-B}$

$-A+B+C=0 \Rightarrow 2B+C=0 \Rightarrow C=-2B \Rightarrow \boxed{C=\frac{2}{3}}$

$A+C=1 \Rightarrow -B+C=1 \Rightarrow -3B=1 \Rightarrow \boxed{B=-\frac{1}{3}}$

$A = \frac{1}{3}$

$$\Rightarrow Y(s) = \frac{\frac{1}{3}}{s+1} + \frac{-\frac{1}{3}s + \frac{2}{3}}{s^2 - s + \frac{1}{4} + \frac{3}{4}}$$

$$Y(s) = \frac{1}{3}\left(\frac{1}{s+1}\right) - \frac{1}{3}\left[\frac{s+2}{(s-\frac{1}{2})^2 + \frac{3}{4}}\right] \Longrightarrow$$

continua después del tema 3