

$$\lambda_3 = -i$$

$$\Rightarrow E_{\lambda_3 = -i} = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} / (A - \lambda_3 I) \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \right\}$$

$$(A - \lambda_3 I) = \begin{pmatrix} 1+i & -1 & -1 \\ 1 & -1+i & 0 \\ 1 & 0 & -1+i \end{pmatrix}$$

$$\Rightarrow \left[ \begin{array}{ccc|c} 1+i & -1 & -1 & 0 \\ 1 & -1+i & 0 & 0 \\ 1 & 0 & -1+i & 0 \end{array} \right] \sim \left[ \begin{array}{ccc|c} i & -1 & -i & 0 \\ 0 & -1+i & 1-i & 0 \\ 1 & 0 & -1+i & 0 \end{array} \right]$$

$$\sim \left[ \begin{array}{ccc|c} i & -1 & -i & 0 \\ 0 & -1+i & 1-i & 0 \\ i & 0 & -i-1 & 0 \end{array} \right] \sim \left[ \begin{array}{ccc|c} a & b & c & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 1-i & 0 \end{array} \right]$$

$$\Rightarrow b = c$$

$$-a = -(1-i)c \Rightarrow a = (1-i)c$$

$$\Rightarrow E_{\lambda_3 = -i} = \left\{ \begin{pmatrix} (1-i)c \\ c \\ c \end{pmatrix} \right\} \Rightarrow v_2 = \begin{pmatrix} 1-i \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \bar{x}_2 = e^{-it} \begin{pmatrix} 1-i \\ 1 \\ 1 \end{pmatrix}$$

$$\bar{x}_2 = [\cos(t) + i \sin(-t)] \begin{pmatrix} 1-i \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \bar{x}_2 = [\cos t - i \sin t] \begin{pmatrix} 1-i \\ 1 \\ 1 \end{pmatrix} = \begin{bmatrix} \cos t - i \sin t - i \cos t - \sin t \\ \cos t - i \sin t \\ \cos t - i \sin t \end{bmatrix}$$

$$\Rightarrow \bar{x}_2 = \begin{bmatrix} \cos t - \sin t \\ \cos t \\ \cos t \end{bmatrix} + i \begin{bmatrix} -\sin t - \cos t \\ -\sin t \\ -\sin t \end{bmatrix}$$

$$\Rightarrow \bar{x} = c_1 x_1 + c_2 x_2$$

$$\left| \begin{pmatrix} x \\ y \\ z \end{pmatrix} \right. = c_1 e^{-t} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} \cos t - \sin t \\ \cos t \\ \cos t \end{pmatrix} + c_3 \begin{pmatrix} -\sin t - \cos t \\ -\sin t \\ -\sin t \end{pmatrix}$$