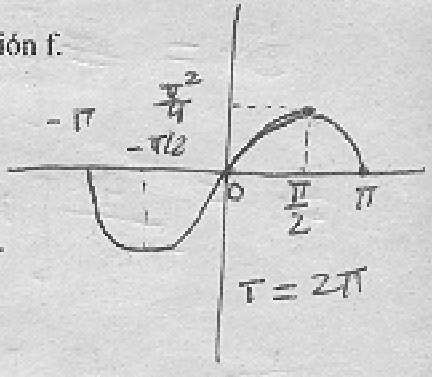
5. (15 puntos)

Con respecto a la función f definida por $f(x) = x(\pi - x)$, $x \in (0, \pi)$:

a. Establezca la correspondiente expansión impar de medio rango de la función f.

b. Determine la suma de la serie numérica $\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{(2n-1)^3}$



$$f(x) = -x^2 + \pi x$$
$$= -x^2 + \pi x - \frac{\pi^2}{4} + \frac{\pi^2}{4}$$
$$f(x) = -\left(x^2 - \pi x + \frac{\pi^2}{4}\right) + \frac{\pi^2}{4}$$
$$f(x) = -\left(x - \frac{\pi}{2}\right)^2 + \frac{\pi^2}{4}, \ x \in (0, \pi)$$

a) $f(x) = \sum_{n=1}^{\infty} b_n \operatorname{sen}\left(\frac{n\pi x}{p}\right)$; $p = \pi$

$$b_n = \frac{2}{p} \int_0^p f(x) \operatorname{sen}\left(\frac{n\pi x}{p}\right) dx$$

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$$b_n = \frac{2}{\pi} \int_0^\pi (-x^2 + \pi x) \operatorname{sen}(nx)\, dx = \frac{2}{\pi}\left[-\frac{2}{n^3}\cos(n\pi) + \frac{2}{n^3}\right]$$

$u = -x^2 + \pi x \Rightarrow du = (-2x + \pi)\, dx$

$dv = \operatorname{sen}(nx)\, dx \Rightarrow v = -\frac{1}{n}\cos(nx)$

$$\int (-x^2 + \pi x) \operatorname{sen}(nx)\, dx = -\frac{(-x^2 + \pi x)}{n}\cos(nx) + \int \frac{(-2x + \pi)\cos(nx)\, dx}{n}$$

$u = -2x + \pi \Rightarrow du = -2\, dx$

$dv = \frac{\cos(nx)}{n} dx \Rightarrow v = \frac{1}{n^2}\operatorname{sen}(nx)$

$$\int (-x^2 + \pi x) \operatorname{sen}(nx)\, dx = \frac{(x^2 - \pi x)\cos(nx)}{n} + \frac{(\pi - 2x)}{n^2}\operatorname{sen}(nx) + \int \frac{2\operatorname{sen}(nx)\, dx}{n^2}$$

$$\Rightarrow \int_0^\pi (-x^2 + \pi x) \operatorname{sen}(nx)\, dx = \left[\frac{(x^2 - \pi x)\cos(nx)}{n} + \frac{(\pi - 2x)}{n^2}\operatorname{sen}(nx) - \frac{2}{n^3}\cos(nx)\right]_0^\pi$$

$$= \left[-\frac{2}{n^3}\cos(n\pi) + \frac{2}{n^3}\right] \Rightarrow$$

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