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An Intergenerational Welfare Analysis in a Small Open Economy Model Between Social Security Systems

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by

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Dedication

This thesis is dedicated to everyone who believes in me, especially those who have helped me through this process and inspired me to continue despite the challenges.

Elvis Jurado

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Abstract

This thesis investigates the long-term macroeconomic and welfare impacts of transitioning from a Pay-As-You-Go to a fully funded pension system, specifically within the Ecuadorian economic context. The study is motivated by financial and demographic challenges that threaten the sustainability of the current pension structure. Understanding the effects of such a transition is essential for informed implementation. The research has two primary objectives: first, to simulate this reform under various economic shocks, particularly changes in oil income and interest rates given that variability in oil revenues directly affects the economy as oil is Ecuador's main source of income; and second, to evaluate how the timing of the changes influences welfare outcomes across generations. The analysis is based on a transition from a Pay-As-You-Go system to a Fully Funded system, allowing for a more flexible response to demographic and fiscal pressures. To achieve this, a calibrated Overlapping Generations model is employed, integrated with a Small Open Economy framework and tailored to Ecuadorian data. This model allows for simulation of the pension reform under different macroeconomic conditions and transition scenarios. Findings suggest that while a fully funded system may increase welfare in the new steady-state equilibrium relative to a PAYG system reflecting the right timing for replacing the social security system under a general equilibrium model positive economic shocks can produce large welfare gains. However, welfare outcomes during the transition period remain highly sensitive to shocks, which in some scenarios can cause net losses for certain generations. The impact varies depending on the type of shock and the timing of reform implementation. These results highlight the importance of timing and economic context when designing pension policy. A poorly timed reform could reduce expected benefits, even if long-term outcomes appear favorable.

Keywords: Overlapping Generations, Welfare, Small Open Economy, Demography, Ageing

JEL Codes: C61, C63, E21, E24, I31

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1 Introduction

The sustainability and efficiency of social security have been discussed in research for decades due to variables, such as demographic transition and fertility, that directly affect an important part of the economy.

One by-product of demographic transition is ageing, which presents itself as a rather complicated issue that The Organization for Economic Co-operation and Development (OECD) countries must constantly face. Needless to say, the viability of social security is simply worrying in the long run, considering that the rate of both, population growth and fertility in the twentieth century, have actually seen a decrease. With this in mind, younger individuals born in later generations are increasingly unable to sustain older ones. We must, therefore, acknowledge that this gap keeps widening as time goes by.

Another point worth considering is that over time, health-care expenditures are expected to increase while future per capita Gross Domestic Product (GDP) growth to decline. When different methods of pension systems are reviewed, the one that is the most commonly used in Latin American countries is the “Pay-As-You-Go” system, which makes it impossible to cope with the problem of demographic transition; for that reason, countries must find a way to escape this challenging cycle.

Experts have come to the conclusion that the given demographic change will cause profound long-lasting economic impacts, both globally and within individual countries. It is vital to consider the optimal timing for changing a country’s pension system; to ensure that societal welfare is maintained; and also bearing in mind that changing a pension system does require numerous reforms, which could potentially become significant barriers.

In general, most Latin American countries have small open economies that have indeed been affected by the demographic transition; this directly influences household consumption, capital, aggregate output, and economic growth in both the short and long run. Due to these phenomena, it is imperative to find a path that points us towards the right time to adjust our social security system, one that is managed more efficiently and ensures only a minimal loss of welfare in the economy.

This study is motivated to find the right moment to change from the current social security system, Pay-As-You-Go, to a new social security system, Fully Funded. To achieve this goal, two primary objectives have to be completed: first, to simulate this reform under various economic shocks, particularly changes in oil income and interest rates, and second, to evaluate how the timing of the reform influences welfare outcomes across generations. Given the central role that oil plays in Ecuador's economy, special attention is paid to how fluctuations in oil prices or extraction levels may affect the performance and long-term sustainability of the proposed pension system.

2 General Literature Review

The literature on Pension Reform Economics is vast, therefore, a comprehensive and analytic review requires dedicated research to meet the field's standards. In order to narrow the scope, there must be a focus on the most relevant papers within the literature.

The most effective way to assess this topic is by using the Overlapping Generation Model (OLG) created by Samuelson (1959) and further refined by Diamond (1965). The OLG model has been employed in various studies, including those examining demographic transition and life-cycle models. In recent years, developing countries have experienced a decrease in fertility and an increase in life expectancy.

The paper by Devriendt W. and Heylen F. (2018) examines the case of Belgium, where the demographic changes have impacted the per-capita growth rate with rather uncertain effects on the economy, the primary response to these effects is the behavior of households and firms. To measure this response, they developed an OLS model for a Small Open Economy that quantifies the net effects in terms of innate abilities with repercussions for education and labor. The results of the model successfully replicate the fundamental evolution of macroeconomic variables and suggest that, in the absence of changes in policy or behavior, the per capita growth could eventually be reduced by 0.25% of GDP annually.

However, if policies are changed, labor supply will be stimulated, which would lead to more hours worked even by less capable workers. The increase in labor could be about 0.29% in the next 25 years, while the public pension expenditures increase by an average of 1.6% Such expenditure increase is one of the major challenges that developing countries could face, particularly as the issue of demographic transition remains pressing.

Bettendorf L. and Heijdra B. (2005) introduced pension reforms in an SOE economy that has a sizable non-traded goods sector. This model separates birth and death probabilities, allows for age dependent labor productivity to take place, and enables a Pay-As-You-Go pension system subject to various shocks, including demographic changes and birth and death probabilities. The conclusion of this research is that the pension payment must be reduced, or the pension age must be increased to cope with the deficit of the pension system that this model uses.

With all this data, the reliability of the PAYG system is in doubt, Stauvermann P. and Kumar R. (2015) showed, using the OLS model in a Small Open Economy (SOE), that the sustainability of PAYG in the presence of a declining population and increasing longevity of the retired generation would only be possible in a distant future, because the growth rates of the pension pay-outs are worryingly lower than the growth rate of wages. One policy that governments should implement is a subsidy for investment in education, to increase wages over time. As a result, the payments into the social security system would not need to be reduced.

However, one significant downside not taken into consideration is the substantial number of freelancing employees and self-employed people in the population who do not contribute to the social security system, something rather common in developing countries, especially in Latin America.

An approach towards the solution of demographic transition is complex due to the various variables affecting public pensions. Although, Kinnunen H. (2008) investigated public pension funding using a Dynamic Stochastic General Equilibrium Macroeconomics Model (DSGE) which allows to identify the distortionary effects of fiscal and pension policies related to aging. The model finds that during the transition to an older population, structural aging costs can be substantially reduced by allowing public funds to smooth out tax responses. Reducing pension pre-funding, when the pace of aging is at its peak, decreases the necessary tax hikes and positively affects labor supply growth. As a result of smaller funding needs, aging actually reduces the growth in labor cost and leads to better employment conditions with faster production growth.

Another approach is to change a country's policy in order to manage the pitfalls in pension systems. The Norwegian pension system is a notable example having undergone a significant reform in 2006 implemented by the government. Haatvedt J. (2008) studied the effects of this reform in the pension funding using both general and partial equilibrium overlapping generation models calibrated to Norwegian data. The results were as follows

- Under the general equilibrium model, the reform led to an increase in welfare and the capital stock in the new steady state with only a marginal change in aggregate labor supply.
- In contrast, the partial equilibrium model showed an increase in welfare and aggregate savings, but a decrease in aggregate labor supply due to the reform.

The evidence reviewed confirms that demographic aging—marked by declining birth rates and rising longevity—undermines the fiscal viability of Pay-As-You-Go pension systems, particularly in small, open Latin American economies. Economic models show that without timely reforms, slower wage growth and rising pension costs will widen funding gaps, while informal employment and limited institutional capacity hinder direct policy transfers from developed countries. The literature thus calls for context-specific reforms balancing welfare and solvency by adjusting retirement age, contribution rates, and investment in human capital. The present dissertation focuses on pension reforms in Latin America, where demographic and economic challenges make sustainable social security especially urgent.

3 Background Models

The first step of this new model is to select and adapt the models we looked at in the last chapter. The models that will be selected must be consistent with our Ecuadorian economy. Additionally, the new model will integrate parts of these models to create a combined framework. This chapter addresses the relevant models and pension systems and is structured as follows:

- Section 3.1 introduces the Overlapping Generations Model, which is suited to reproduce certain features of the population, such as demographic growth.
- Section 3.2 defines Social Security Systems, which are necessary to understand the timing and nature of transitions between them.
- Section 3.3 introduces the Small Open Economy because it better adapts our economy into the final model.

3.1 Overlapping Generations Model

One important model to take into consideration in this research is the Overlapping Generations Model (OLG) which was developed by Allais (1947), Samuelson (1958), and Diamond (1965). This fundamental model, well-known in the field of macroeconomics, operates as follows: at any given time, individuals from different generations are alive and maybe trading with one another. Each generation interacts with other generations at different periods of its life. The OLG model is extensively studied because it allows for the analysis of the aggregate implications of life-cycle saving. These savings become capital stock as individuals need to finance their consumption during retirement. A key result of the OLG model is that the competitive equilibrium may not be Pareto optimal, as individual savings may be over-accumulated.

The simplest OLG model is the two-period version, where individuals live for only two periods. In this model, individuals interact in the market at different stages of their life cycles; a young person interacts with an older person, and later, as the young person ages, the interaction shifts to mostly younger individuals. Typically, this economy is composed of two cohorts or generations, often referred to as the young and the old.

3.1.1 The Decentralized Equilibrium

In the economy, various subjects interact, including individuals and firms. In this case, individuals live for only two periods: they are born at time t, and consume C_t in period t, and C_{t+1} in the period t+1 with a utility function of:

$$u(c_t) + (1 + \vartheta)^{-1}u(c_{t+1}),$$

where,

$$\vartheta \geq 0, u'(\cdot) > 0, u''(\cdot) < 0.$$

It is important to note that individuals work only in the first period of their lives, supplying inelastically one unit of labor and earning a real wage of w_t. They only consume a portion of their salary in the first period and save the remainder to consume during the second period of retirement. The number of individuals born at time t and working in period t is N_t. Also, the population grows at a rate n, so $N_t = N_0(1 + n)^t$. On the other hand, firms act competitively and use constant returns technology, represented by $Y = F(K, N)$. The goal of each firm is to maximize profits, taking the wage rate, w_t, and the rental rate on capital r_t as given. Next, we will examine the optimization problems of individuals and firms and derive the market equilibrium.

3.1.2 Individuals

The maximization problem for an individual born at time t is:

$$\max\ u(c_t) + (1 + \vartheta)^{-1}u(c_{t+1})$$

subject to

$$c_t + s_t = w_t,$$

$$c_{t+1} = (1 + r_{t+1})s_t$$

where w_t is the wage received in period t, and r_t +1 is the interest rate on savings from period t to period t+1. In the final period, individuals consume all their resources, encompassing both interest and principal.

The first-order condition for the maximization problem is expressed as:

$$u'(c_t) - (1 + \vartheta)^{-1}(1 + r_{t+1})u'(c_{t+1})$$

Substituting for C_t and $C_t + 1$ in terms of savings s, wages w, and interest rate r yields the following saving function:

$$s_t = s(w_t, r_{t+1}),$$

$$0 < s_w < 1,$$

$$s_r \lesseqgtr 0.$$

3.2 Social Security Systems

It is well-known that Social Security System affects both capital accumulation and the welfare of an economy. Due to this, social security programs were introduced to ensure a minimum level of income in retirement, as individuals might not save enough for their old age. Additionally, any program that impacts people's income will have repercussions on savings and capital accumulation.

Individuals make a social security contribution while they are young and receive payments in their old age from the social security system. Let d_t denote the contribution of a young person at time t and b_t denote the benefit received by an old person in period t.

There are two fundamentally different methods to run a social security system:

Fully Funded System: In this system, the contributions of the young at time t are invested and returned with interest at time t+1 to the then-old. For this case, $b_t = (1 + r_t)d_{t-1}$ where r_t is the rate of return on social security contributions.

Pay-As-You-Go System: This system acts like an unfunded scheme where current contributions made by the young are directly transferred to the current old. In this case, $d_t = (1{+}n)d_t$ and the rate of return on the contributions is n.

3.2.1 Fully Funded System

Governments that use a Fully Funded System raise d_t in contributions from the young at period t, invest d_t as capital, and pay $b_t = (1 + r_t)d_{t-1}$ to the old, whose contributions were invested at period t-1. For the use of this method, the first-order conditions of the individuals and the equation for goods market equilibrium become:

$$u'[w_t - (s_t - d_t)] = (1 + \vartheta)^{-1}u'[(1 + r_{t+1})(s_t + d_t)],\ s_t + d_t = (1 + n)k_{t+1}$$

We can see that if k_t is the solution for the first system, it is also the solution for the second. The previous statement implies $d_t < (1 + n)k_{t+1}$ which means that social security contributions do not exceed the amount of saving that would otherwise have occurred. A clear conclusion is that fully funded social security does not affect total savings and capital accumulation. Additionally, if d_t increases, it will be offset by a decrease in private savings, so that the total, $s_t + d_t$ is the same as the previous level of s_t.

The result of this method provides a rate of return equal to the rate of private savings. Due to this, individuals are indifferent to who does the saving; they are only concerned about the rate of return. Therefore, consumers compensate through private savings for whatever savings the social security system does on their behalf.

3.2.2 Pay-As-You-Go System

Governments that use a Pay-As-You-Go System must manage savings quite differently; in this case, the first-order conditions of the individuals and the equation for goods market equilibrium become, using the next expression $b_{t+1} = (1 + n)d_{t+1}$:

$$u'[w_t - s_t - d_t] = (1+\vartheta)^{-1}u'[(1+r_{t+1})s_t + (1+n)d_{t+1})],\ s_t = (1+n)k_{t+1}$$

In this method, the rate of return on social security savings is n rather than r, because in each period, there are more people alive and working who contribute to the social security system. This system is a pure transfer scheme with no savings, and the only source of capital for the economy is private savings s_t.

It is important to consider the effects of social security on wages and interest rates in relation to private savings. To analyze this, we differentiate and assume $d_t = d_{t+1}$ yielding the following results:

$$\frac{ds_t}{dd_t} = -\frac{u''_y + (1+\vartheta)^{-1}(1+n)u''_o}{u''_y + (1+\vartheta)^{-1}(1+r_{t+1})u''_o} < 0,$$

Here, $ds_t/dd_t \lesseqgtr 1$, depending on whether $n \lesseqgtr r$. The signs indicate that the wage rate and interest rate are held constant. An unavoidable consequence of the contributions to social security is a decrease in private savings, which directly affects the reduction of capital, and wages, and leads to an increase in the interest rate that has an ambiguous effect on the economy.

3.3 Small Open Economy

The world economy comprises many small economies that interact with each other; with each transaction having a negligible impact on the global economy. This model assumes that each Small Open Economy shares identical preferences, technology, and market structures.

Most macroeconomic interactions in a small open economy are related to the inter-temporal trade, which involves the exchange of resources across time. Inter-temporal trade is measured by the current account of the balance of payments.

An adaptation of Irving Fisher's (1930) model will be used for the case of a small open economy that consumes a single good over two periods: the young and the old.

3.3.1 Consumer's Problem

In this model, an individual i maximizes lifetime utility U^i_1 which depends on consumption levels in both periods c^i.

$$U^i_y = u(c^i_y) + \beta u(c^i_o), \quad 0 < \beta < 1.$$

Where β is the subjective discount factor or time-preference factor that measures the individual's impatience to consume. As usual, the assumptions for the utility function $u(c^i)$ are: $u'(c^i) > 0$ strictly increasing in consumption, and $u''(c^i) < 0$ strictly concave.

Let y^i denote the individual's output and r the real interest rate in the world capital market on date 1. The lifetime budget constraint for consumption is:

$$c^i_y + \frac{c^i_o}{1+r} = y^i_y + \frac{y^i_o}{1+r}$$

This constraint restricts the present value of consumption spending to be equal to the present value of output. Since output is perishable, it cannot be stored for later consumption.

The first-order condition for the previous problem is:

$$u'(c^i_y) = (1 + r)\beta u'(c^i_o),$$

Which is known as the intertemporal Euler equation. This equation can be interpreted as follows: at utility maximum, the consumer cannot benefit from feasible shifts in consumption between periods. Each individual i's optimal consumption plan is determined by combining the first-order condition with the intertemporal budget constraint. An important case arises when $\beta = \frac{1}{1+r}$, because the subjective discount factor equals the market discount factor. In this scenario $u'(c^i_y) = u'(c^i_o)$ which implies that the consumer has a flat lifetime consumption path, $c^i_y = c^i_o$. Thus, the consumption in both periods will be:

$$c^i = \frac{[(1+r)y^i_y + y^i_o]}{2+r}$$

4 The Benchmark Model

The main objective of this model is to analyze and research the long-run macroeconomics and welfare levels in two different pension reforms using a model calibrated to Ecuadorian historical data. Additionally, the model combines the Overlapping Generations (OLG) and Small Open Economy (SOE) frameworks, where the main economic assumption that is taken into the model is that each generation lives for only 30 years and the range of analysis will span eight generations.

This chapter addresses the two possible pension reforms and their welfare effects and is structured in the following way:

- Section 4.1 introduces the environment of the model in each area that participates in the economy. This model is fitted to reproduce certain features of the Ecuadorian economy such as population growth, labor technology growth, agent consumption, applied transfers, and the social security systems (Pay-As-You-Go and Fully Funded) that alter the distribution of transfers.

- Section 4.2 defines equilibrium in the entire economy and the methods used to calculate different transfers and determine welfare levels.

- Section 4.3 calibrates the model parameters using Ecuadorian historical data.

4.1 The Environment of the Model

Demographics

In each period t, a new generation is born; the duration of a generation is 30 years. Individuals grow at a rate η_t per period. The population growth in this model is given by the following equation:

$$N_{t+1} = N_t(1 + \eta_{t+1}) \quad (1)$$

Technology

A representative firm uses a linear Cobb-Douglas production function that uses only labor as input and labor that augments technological growth to produce output. The function is represented by:

$$Y_t = A_t L_t \quad (2)$$

Where A_t is the labor augmenting technology factor and L_t is the labor input such as hours of work. The labor technology factor is determined by:

$$A_{t+1} = A_t(1 + g_{t+1}) \quad (3)$$

Where g is the growth rate.

The next maximization linear problem gives the general equilibrium of this representative firm:

$$max\ \{A_t L_t - w_t L_t\}\ , \quad (4)$$

In this equilibrium, capital is not part of the maximization problem. The process of obtaining the equation is also in the appendix. Here w_t is the wage rate and the equilibrium of the linear problem yields the result:

$$w_t = A_t \tag{5}$$

Households' problem

Individuals derive utility from the consumption during both their youth and old age; furthermore, the population does not have a bequest motive. Individuals seek to maximize their utility. As a consequence, each individual born at time t in different generations faces the following problem:

$$maxU_t = \frac{(C_t^y)^{1-\gamma} - 1}{1-\gamma} + \beta E_t \frac{(C_{t+1}^o)^{1-\gamma} - 1}{1-\gamma} \tag{6}$$

Subject to a set of budget constraints:

$$C_t^y + a_{t+1} = w_t - \tau_t \tag{7}$$

$$C_{t+1}^o = (1 + r_{t+1})a_{t+1} + T_{t+1} \tag{8}$$

In the previous equations β is the discount factor, C_t^y is the consumption of the youth, C_{t+1}^o is the consumption of the old ones. Also, a_{t+1} is the assets of each individual, r_{t+1} is the interest rate, T_{t+1} is the lump-sum transfer receives when individuals are old and comes from the social security payroll tax rate τ_t paid during their youth. It is also clear that hours of work and consumption cannot be negative; therefore, they must satisfy the following conditions:

$$a_{t+1} \in [0, 1]$$

$$C_t^y \geq 0$$

$$C_{t+1}^o \geq 0$$

The maximization problem of the households gives the following equation of Euler:

$$(C^y_t)^{-\gamma} = E_t\beta(1 + r_{t+1})(C^o_{t+1})^{-\gamma} \tag{9}$$

Lump sum transfers

Since agents live in only one generation at a time, all the resources are consumed within the same period of life, with no bequest to the next generation. As a result, the government uses oil income to distribute lump sum transfers to agents alive in their old age at no cost. With this in mind, the government transfers the income in the following form:

$$N_t\tau_t + QP^{oil}_t + IT = N_{t-1}T_t \tag{10}$$

$$\tau_t = \tau w_t \tag{11}$$

Where QP^{oil}_t represents oil income at time t; also, N_t, N_{t-1} refer to the population sizes of the young and old, respectively. IT represents the internal transfers that are different from oil income. Additionally, τ_t is given by a payroll tax τ on the salaries w_t in each period t. It is important to clarify that the transfers depend on the type of social security system applied in the economy.

Social Security

An agent who works throughout his life must retire at some point t, and receive pension benefits T_{t+1}, calculated as a fraction of his social security payroll and the oil income.

In the Pay-As-You-Go system the equation that illustrates the transfers to agents is given by:

$$T_{t+1} = \frac{N_{t+1}\tau_{t+1} + QP^{oil}_{t+1} + IT}{N_t} \tag{12}$$

The transfers T_{t+1} in this Social security system depend on social security payroll the entire population $N_{t+1}\tau_{t+1}$, the oil income QP^{oil}_{t+1} and IT internal transfers divided by the number of agents N_t.

On the other hand, the Fully-Funded system is calculated as follows:

$$T_{t+1} = (1 + r_{t+1})\left(\tau_t + \frac{QP^{oil}_{t+1} + IT}{N_t}\right) \tag{13}$$

In this system, transfers under the Fully Funded system T_{t+1} depend on the social security payroll and oil income per capita, adjusted to the future by an interest rate r_{t+1}.

4.2 Equilibrium

An equilibrium for this economy, given the demographic growth structure consists of sequences over different generations of two social security tax rates τ_t, lump-sum transfers T_{t+1}, households allocations $[C^y_t, C^o_{t+1}, a_{t+1}]$, the factor for the firm A_t and factor prices w_t, r_{t+1} given by:

1. Given the two payroll tax rates, lump-sum transfers, and factor prices, households can solve their optimization problem.

2. Given factor prices, the representative firm's optimization problem can be solved.

3. The equilibrium is defined as the point where all markets clear:

- The labor market equilibrium (5) is:

$$A_t = w_t$$

- The household equilibrium (9) is:

$$(C^y_t)^{-\gamma} = E_t \beta (1 + r_{t+1})(C^o_{t+1})^{-\gamma}$$

- Combining the transfers as the Pay-as-You-Go system (12) and the Euler equation for consumption (9) leads to the following expression:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t \beta (1 + r_{t+1})^{1-\gamma} (\hat{a}_{t+1} + \frac{(1 + \eta_{t+1})(1 + g_{t+1})}{1 + r_{t+1}} (\tau + Z_{t+1}))^{-\gamma} \tag{14}$$

- Similarly, combining the transfers under the Fully-Funded system (13) and the Euler equation for consumption (9) results in the following:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t \beta (1 + r_{t+1})^{1-\gamma} (\hat{a}_{t+1} + (\tau + Z_t))^{-\gamma} \tag{15}$$

- Where:

$$Z_t \equiv \frac{QP^{oil}_t + IT}{N_t A_t} \tag{16}$$

$$a_{t+1} = \hat{a}_{t+1} A_t \tag{17}$$

The preceding equations are re-expressions of the oil income and the augmenting technology.

4. Using a numerical solution of the algorithm "By-section", where we aim to find the zero of a one-dimensional function that represents the transition between the Pay-as-

You-Go and Fully-Funded systems. The equation is given by:

$$(1-\tau-\hat{a}_{t+1})^{-\gamma} = (1+\pi)^2 E_t \beta (1+r_{t+1})^{1-\gamma}\left(\hat{a}_{t+1} + \frac{(1+\eta_{t+1})(1+g_{t+1})}{1+r_{t+1}}(\tau + Z_{t+1})\right)^{-\gamma}$$
$$+\pi E_t \beta (1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1} + (\tau + Z_t))^{-\gamma} + (1-\pi)\pi E_t \beta (1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1})^{-\gamma} \quad (18)$$

5. The variables r_{t+1}, η_{t+1}, g_{t+1} and Z_{t+1} are subject to different random shocks, as given by:

$$r_{t+1} = (1-\vartheta_r)r^* + \vartheta_r r_t + \sigma_r \epsilon^r_{t+1} \quad (19)$$

$$\eta_{t+1} = (1-\vartheta_\eta)\eta^* + \vartheta_\eta \eta_t + \sigma_\eta \epsilon^\eta_{t+1} \quad (20)$$

$$g_{t+1} = (1-\vartheta_g)g^* + \vartheta_g g_t + \sigma_g \epsilon^g_{t+1} \quad (21)$$

$$Z_{t+1} = (1-\vartheta_Z)Z^* + \vartheta_Z Z_t + \sigma_Z \epsilon^Z_{t+1} \quad (22)$$

6. We must examine how the welfare analysis is derived, considering that we assumed our economy is governed by a central planner who discounts the utility of each generation at a rate R. Additionally, we must assume that the utility of both current and future generations is the primary concern of the planner, who seeks a social welfare function that represents the sum of the utilities of all generations over a specific period. In a Benthamite fashion, he weights utility by the size of each generation for this reason, I will employ the method outlined in Blanchard,O.& Fisher,S. (1993). Which yields the Benthamite Equation:

$$U = (1+\vartheta)^{-1}u(c2_o) + \sum_{t=0}^{T=1}(1+R)^{-t-1}[u(c1_t) + (1+\vartheta)^{-1}u(c2_{t+1})] \quad (23)$$

4.3 Calibration

The model requires the calibration of several parameters that will help define the economy and enable a numerical solution.

Table 1: Parameters

Parameters Requiring Calibration										
γ	β	τ	η	ϕ_r	ϕ_g	ϕ_Z	σ_r	σ_g	σ_Z	π

Coefficient of Relative Risk Aversion

The parameter γ measures the degree to which an agent dislikes risk relation to their current wealth level. An individual whose gamma value is high means the individual is more risk-averse. The value for γ is determined from the lectures, where a common value is near 2.

Subjective Discount Factor

The parameter β characterizes the impatience of agents, reflecting their preference for current consumption over future consumption. As a result, future benefits are valued less than the present ones. In standard economic models, the average value of β is typically set at 0.97. Patnaik, A., Venator, J., Wiswall, M., & Zafar, B. (2022)

Social Security Rate

I set $\tau = 0.2$ to approximate the social security tax rate in Ecuador, as the combined employee and employer tax is, on average, close to this value.

Population Growth Rate

The rate of growth in the number of agents in a period t is defined as η. To calibrate η, I reference the average annual population growth of Ecuador (2000-2023) from World Health Organization data, which is approximately 1.1% with a projected increase of 24% by 2050.

Interest Rate

The interest rate is denoted by r. To calibrate r, I estimate an Ar(1) model using interest rate data (2000-2023) from the Central Bank of Ecuador, as follows:

$$r_{t+1} = \phi_r r_t + \epsilon^r_{t+1} \tag{24}$$

The parameter ϕ_r allows me to fit the interest rate, and the estimated value based on the data is $\phi_r = 0.7905$, with a standard deviation $\sigma_r = 0.2119$.

Labor Augmenting Technology

The rate of growth in labor technology is defined by g. To calibrate g, I estimate an Ar(1) model using the inflation data series (2003-2023) from the Central Bank of Ecuador, which is given by:

$$g_{t+1} = \phi_g g_t + \epsilon^g_{t+1} \tag{25}$$

The result is the value of the parameter ϕ_g that allows me to fit the g parameter. Using the data, I estimate $\phi_g = 0.7521$ with a standard deviation of $\sigma_g = 0.0854$.

Oil Income

Oil income is defined by Z in equation (16). To calibrate Z, I estimate an Ar(1) model using the oil income data series (1995-2023) from the Central Bank of Ecuador, given by:

$$Z_{t+1} = \phi_Z Z_t + \epsilon^Z_{t+1} \tag{26}$$

The result is the value of the parameter ϕ_Z that allows me to fit the Z parameter. Using data, I estimate $\phi_Z = 0.8104$ with a standard deviation of $\sigma_Z = 0.1353$.

Incidence Probability

The incidence probability is challenging to determine, as the exact timing of system changes is uncertain. In this case, an intermediate probability of $\pi = 0.5$ is selected, which is equivalent to the probability of tossing a fair coin.

To summarize the parameter values in this section.

Table 2: Parameters Calibrated

Parameters Requiring Calibration										
γ	β	τ	η	ϕ_r	ϕ_g	ϕ_Z	σ_r	σ_g	σ_Z	π
2	0.97	0.2	1.1%	1	0.397	1	0.011	0.154	0.06	0.5

4.4 Numerical Simulation

The competitive equilibrium is defined in section 4.2, where different methods of transfers are considered, depending on the social security tax selected. When reassembling some growth variables, the set of equations that describe the solution are affected in the following ways:

- The social security transfers.
- The consumption of households.
- The transfers from governments.
- The welfare equation.

Social Security Transfers

Pension benefits that show the change between the Pay-as-You-Go and Fully-Funded systems paths are given in equation (18):

$$(1-\tau-\hat{a}_{t+1})^{-\gamma} = (1+\pi)^2 E_t\beta(1+r_{t+1})^{1-\gamma}\left(\hat{a}_{t+1} + \frac{(1+\eta_{t+1})(1+g_{t+1})}{1+r_{t+1}}(\tau + Z_{t+1})\right)^{-\gamma}$$
$$+\pi E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1} + (\tau + Z_t))^{-\gamma} + (1-\pi)\pi E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1})^{-\gamma}$$

This equation represents the probability of changing the social security system and will allow us to find the variables $\hat{a}_{t+1}$, r, g, Z, n.

On the other hand, pension benefits that represent the new social security system under Fully-Funded scenario are given in equation (15):

$$(1-\tau-\hat{a}_{t+1})^{-\gamma} = E_t\beta(1+r_{t+1})^{1-\gamma}(\hat{a}_{t+1} + (\tau + Z_t))^{-\gamma}$$

This equation represents the new system and allows us to find the updated values for the variables $\hat{a}_{t+1}$, r, g, Z, and n under the new scenario. Both pension benefit equations are solved in the appendix.

Optimal Consumption of Households

As we know, the equilibrium of consumption is given by the Euler equation (9):

$$(C^y_t)^{-\gamma} = E_t\beta(1+r_{t+1})(C^o_{t+1})^{-\gamma}$$

This equation shows the equality between current and future consumption. The Euler equation is solved in the appendix.

Although equilibrium between both consumptions is achieved, the equations to calculate each one are necessary for the results. The process to obtain these equations is detailed in the appendix. The current and future consumptions are given by equations (7) and (8):

$$C^y_t = A_t(1-\tau) - a_{t+1}$$

$$C^o_{t+1} = (1 + r_{t+1})a_{t+1} + T_{t+1}$$

The Transfers

The transfers problem is solved in the appendix. The re-expression of transfers as functions of endogenous variables is derived from the two social systems.

The Fully-Funded transfer is given by:

$$T_{t+1} = (1 + r_{t+1})(\tau + Z_t)A_t \tag{27}$$

The Pay-as-You-Go transfer is given by:

$$T_{t+1} = (1 + n_{t+1})(1 + g_{t+1})(\tau + Z_t)A_t \tag{28}$$

The Welfare Equation

The welfare equation in our model, which comes from the Command Optimum in the Lecture of Macroeconomics book is given by:

$$U = \left[\frac{(C^y_t)^{1-\gamma} - 1}{1-\gamma} + \beta\frac{(C^o_{t+1})^{1-\gamma} - 1}{1-\gamma}\right] N_t \tag{29}$$

This equation measures the welfare level of different generations and will indicate which scenario is better at the societal level. Additionally, it will help us determine the optimal time to transition between systems.

4.5 Numerical Algorithm

The model that combines the OLG and SOE models is characterized by equations (6)-(8), (17)-(22), and (27)-(29). To solve the model I apply the following process:

1. Select the endogenous variables (r, n, g, z, $\hat{a}_{t+1}$, a_{t+1}, C^y_t, C^o_{t+1}, T_{t+1}, U_t, U).

2. The equilibrium is defined by equations (6)-(8), (17)-(22), and (27)-(29)

3. This system of eleven equations in eleven endogenous variables is solved using using MATLAB.

4. Iterate 1000 times and analyze various scenarios that depend on the temporal evolution of systems (2 or 6) and exogenous variables such as oil price, interest rate, and technology growth rate.

5. Select the optimal welfare value from the average welfare obtained across different iterations and generations in the code.

5 Results

In this thesis, I aim to evaluate different scenarios that directly affect the agent's consumption behavior. Based on these scenarios, the resulting levels of societal welfare will be assessed in order to identify the most favorable outcomes. I examine six scenarios in which the timing of transition between social security systems varies. Specifically, the system change occurs at either T=2 or T=6. For each transition period, the economy is subjected to different exogenous shocks: an increase in the oil income, a decrease in the interest rate, and a combination of both shocks. The figures below illustrate the outcomes of these scenarios:

Increase in Oil Income and Change in T=2

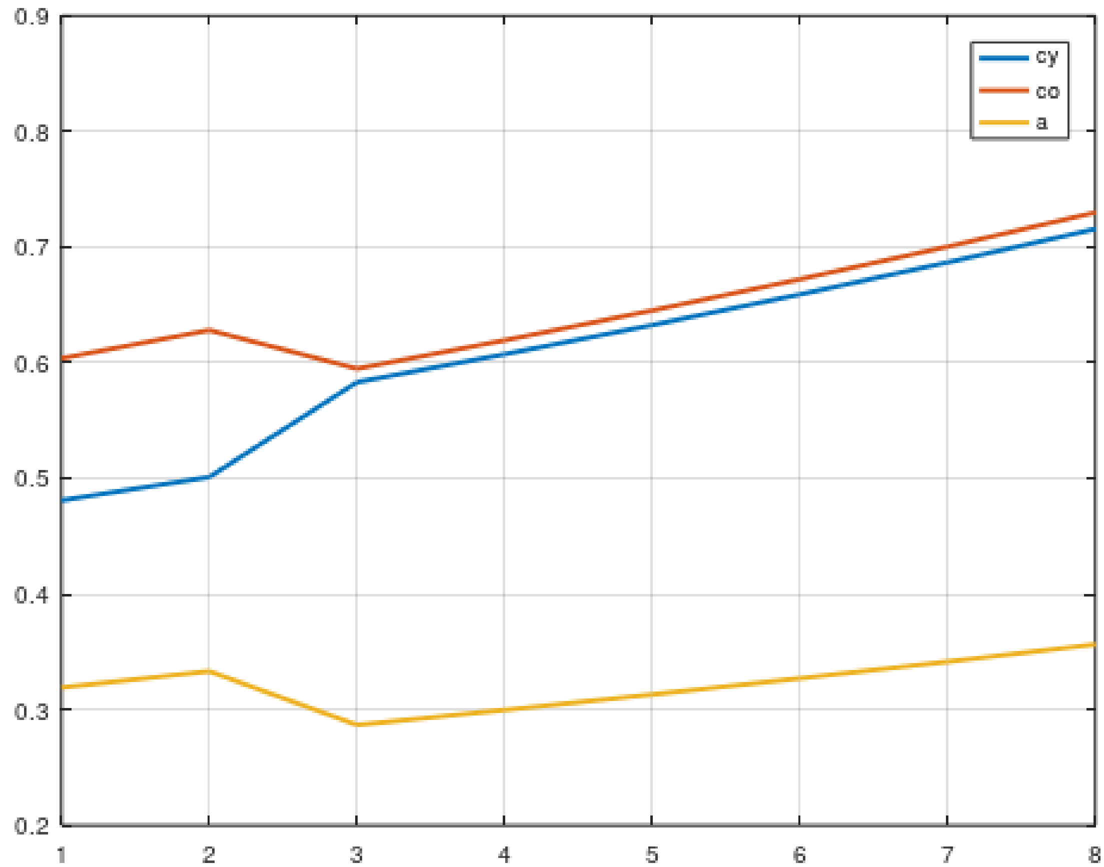

Figure 1: Increase of Z - Jump in T=2

With an increase in the price of oil and a change in the social security system at T=2, the consumption of young agents increases from 0.48 to 0.58. This occurs because higher disposable income enables them to shift some future consumption to the present. In contrast, the consumption of elderly agents decreases from 0.63 to 0.59, as they receive lower benefits following the system change. At T=3, the consumption paths of both groups converge and stabilize, indicating a smoothing effect over time.

Decrease in Interest Rate and Change in T=2

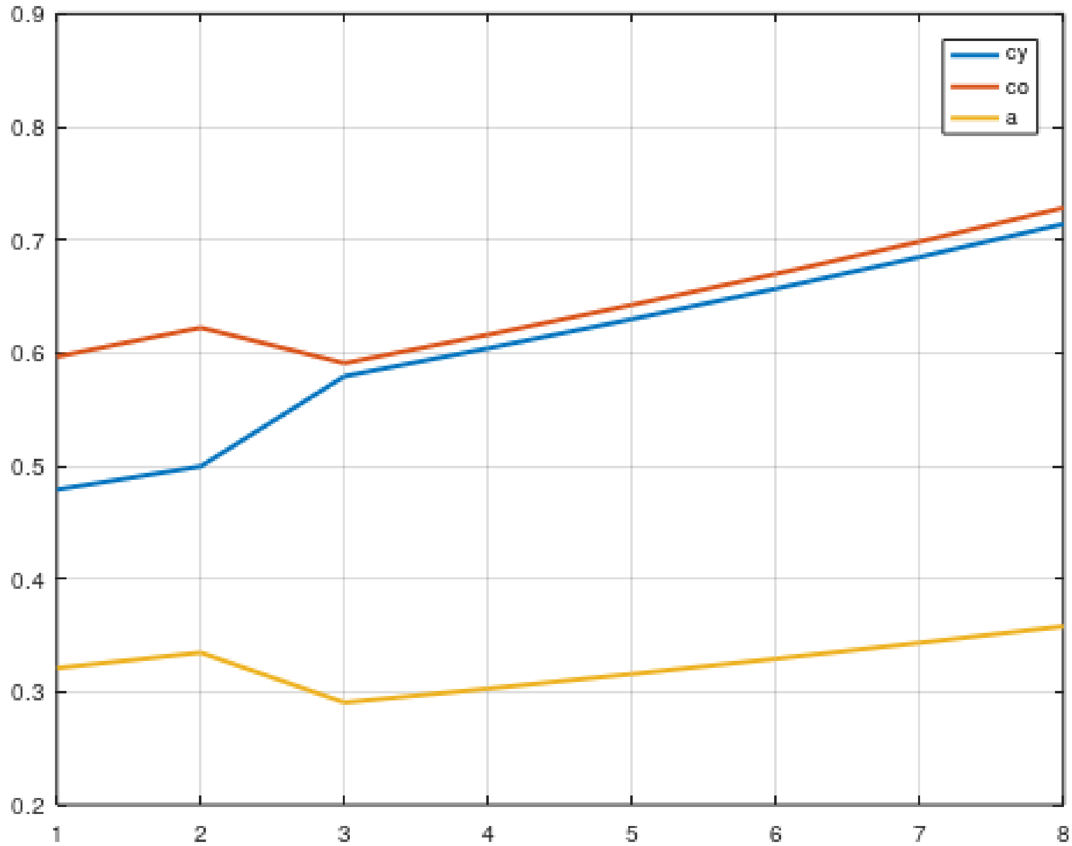

Figure 2: Decrease of R - Jump in T=2

In this scenario, a system change at T=2 coincides with a decrease in the interest rate. As a result, the consumption of young people increases from 0.50 to 0.57, due to a lower incentive to save for the future. Meanwhile, the consumption of the elderly decreases from 0.63 to 0.59, since the reduced interest rate yields lower returns on their savings. From T = 3, consumption levels of both groups smooth out and converge.

Increase in Oil Income and Decrease of Interest rate and Change in T=2

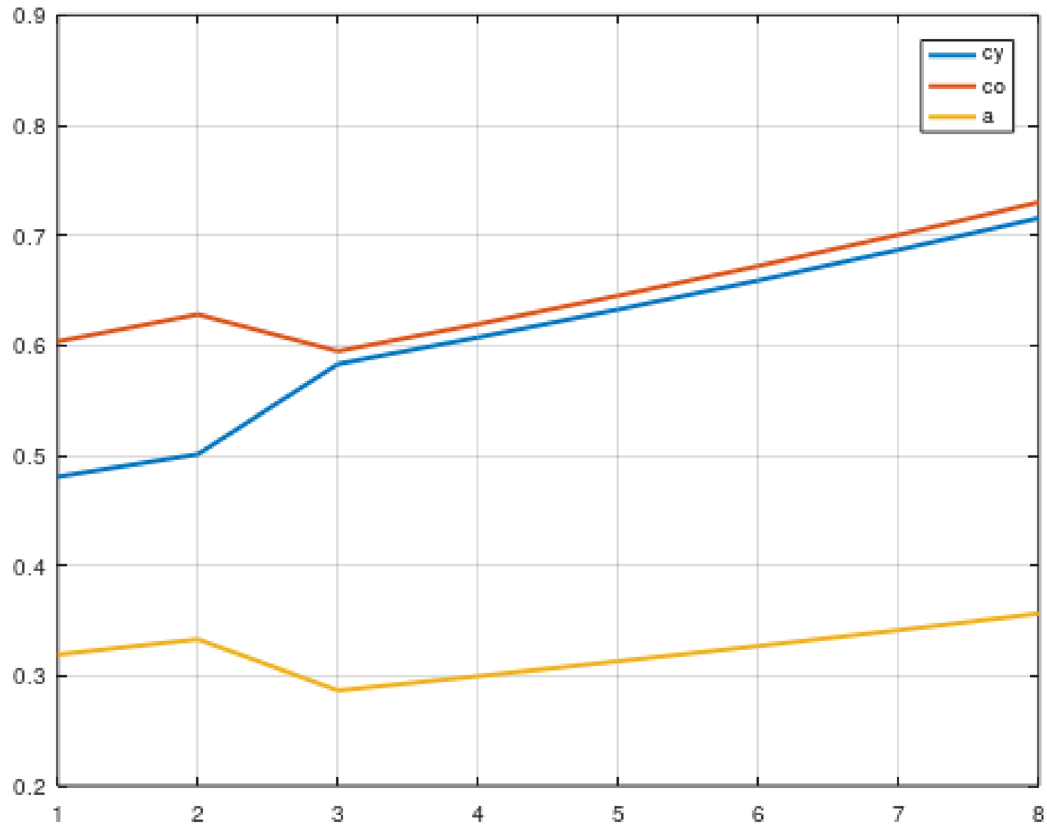

Figure 3: Increase of Z - Decrease of R - Jump in T=2

This scenario combines an increase in oil income and a decrease in the interest rate, with a system change occurring at T=2. The consumption of young people increases from 0.50 to 0.58, encouraged by the dual effect of higher income and the lower returns to saving, which incentivize present consumption. On the other hand, the consumption of elderly agents declines from 0.64 to 0.60, as both their savings and redistributed resources are negatively affected. From T=3, the consumption levels of both groups begin to smooth out and nearly overlap.

Increase in Oil Income and Change in T=6

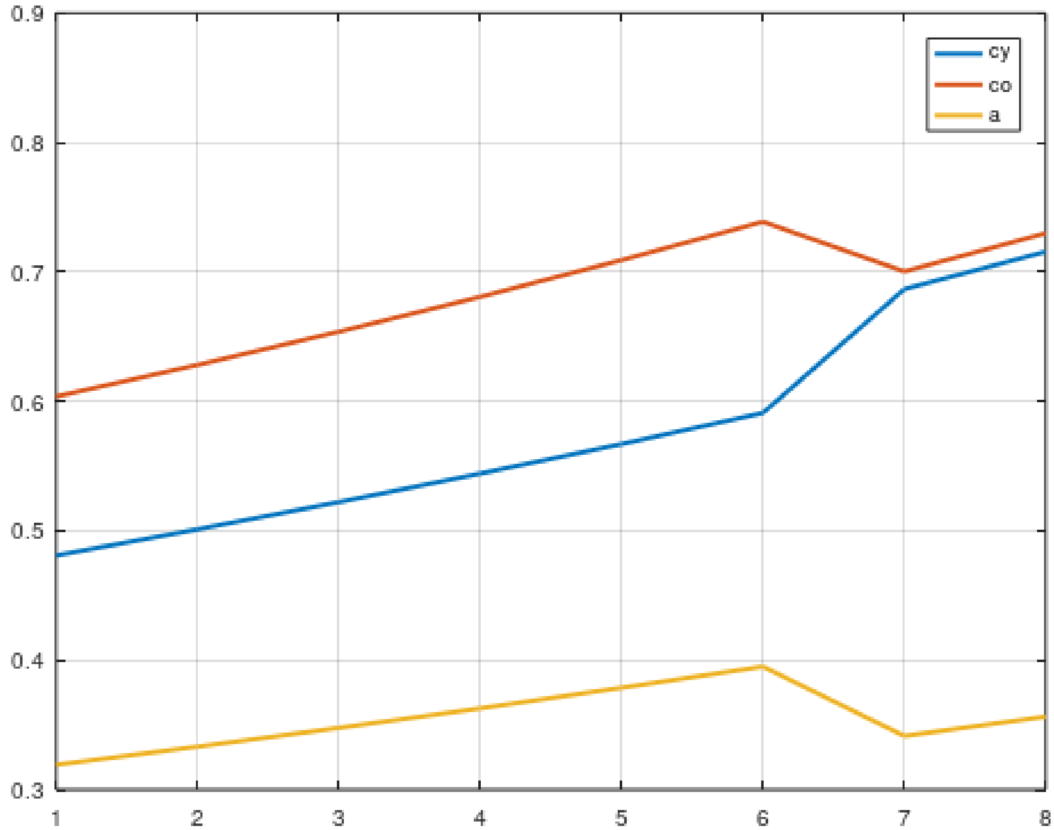

Figure 4: Increase of Z - Jump in T=6

In this scenario, the oil income increases while the system change is delayed until T=6. The consumption of young people increases, and it shows that their consumption increases from 0.59 to 0.78, as the additional income leads them to exchange future consumption for present consumption. In contrast, the consumption of elderly agents decreases from 0.74 to 0.7, since they receive fewer resources following the redistribution that comes with the new system. From T=7, both groups'consumption paths stabilize permanently.

Decrease of Interest Rate and Change in T=6

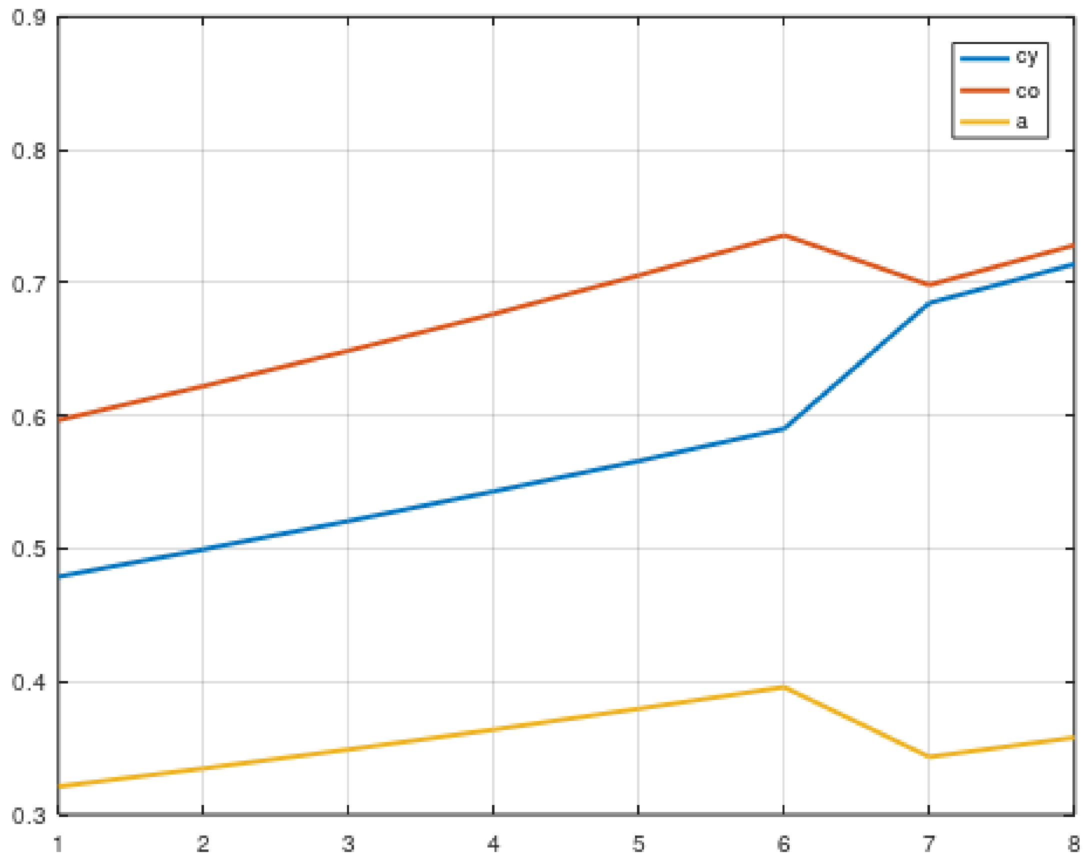

Figure 5: Decrease of R - Jump in T=6

Here, a decrease in the interest rate is combined with a system change at T=6. The consumption of young agents increases from 0.59 to 0.68, driven by the disincentive to save money, which encourages present consumption. Meanwhile, the consumption of elderly people drops from 0.74 to 0.70, as their savings generate lower returns. From T=7, consumption levels settle out permanently.

Increase in Oil Income and Decrease of Interest Rate and Change in T=6

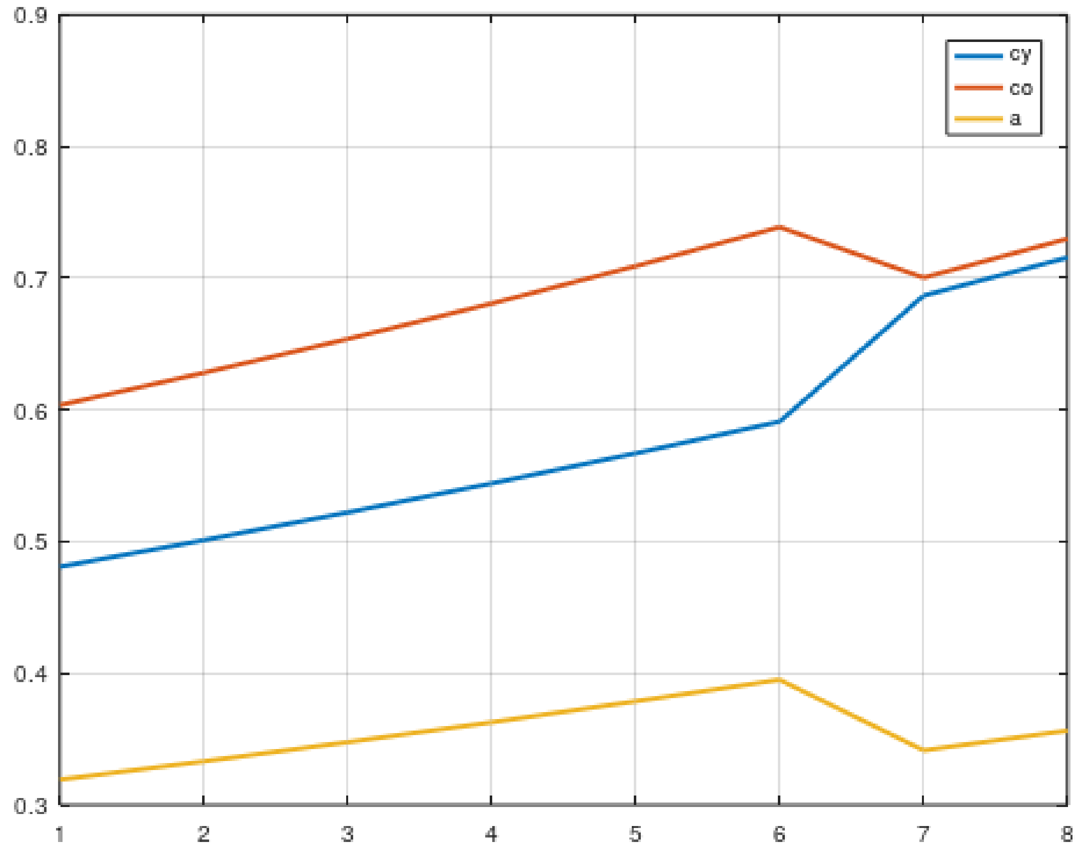

Figure 6: Increase of Z - Decrease of R - Jump in T=6

In the final scenario, both shocks are introduced at T=6, alongside the system change. The consumption of youth agents increases from 0.59 to 0.68 due to the combined effects of increased oil income and lower incentives to save. In turn, elderly consumption falls from 0.74 to 0.70, as the redistribution under the new system and lower returns on savings reduce their capacity to consume. From T=7, both groups experience stable, smoothed consumption paths.

Welfare analysis

In the previous scenarios, I used the Benthamite equation to measure welfare levels, following the method outlined by Blanchard,O.& Fisher,S. (1993). As mentioned earlier, six possible scenarios are analyzed.

Table 3: Welfare Values

Scenarios	T=2	T=6
Increase P(z)	-10,261	-10,479
Decrease R	-10,362	-10,543
Increase P(z) and Decrease R	-10,234	-10,439

Different scenarios can reflect various shocks that may affect the welfare, especially when we take into account the most important shocks in the Ecuadorian economy. This section evaluates welfare impacts under three shock scenarios over two time periods (T = 2 and T = 6), where we consider changing the social security system between Pay-As-You-Go and a Fully funded model. The Benthamite equation is used to quantify aggregate utility. Negative values reflect welfare losses. The scenarios examine: (1) an oil price increase, (2) an interest rate decrease, and (3) both shocks occurring simultaneously.

The first scenario reflects a shock caused by an oil price increase. Under this shock, the change between systems could be implemented at either T=2 or T=6. The welfare values in each period are -10.261 and -10.479, respectively.

The second scenario reflects a decrease in the interest rate. If the interest rate decreases, we could change our social security system at time T=2 or T=6, with the welfare values of -10.362 and -10.543 in each period.

The third scenario reflects the combination of both shocks. If both shocks occur in the economy, the change could be made at either time, with welfare values of -10.234 at time T=2 and -10.439 at time T=6.

If we compare the scenarios at times T=2 and T=6, the third scenario, representing the combination of both shocks, results in the lowest welfare values. Based on an analysis of times and scenarios, the third scenario with a change in the social security system at time T=2 shows the lowest welfare value of -10.234 among all the possible combinations.

6 Conclusion

This thesis has examined the long-run macroeconomic and welfare effects of introducing a fully funded pension system in an economy that will replace the older scheme, where the Pay-As-You-Go (PAYG) system is the primary mechanism, replicating key features of the Ecuadorian economy. It was found that the introduction of a fully funded pension system results in welfare gains for agents born into the new long-run equilibrium, compared to a scenario in which the PAYG system remains in place. However, the extent of these welfare improvements varies depending on the economic shocks affecting the economy. This analysis was conducted using an Overlapping Generations (OLG) model within a Small Open Economy framework calibrated to Ecuadorian data.

This study considered the effects of three macroeconomic shocks:

- An oil price increase.
- An interest rate decrease.
- Both occurring simultaneously.

The increase in household welfare at the long-run equilibrium depends on whether and how these shocks occur. A rise in oil prices directly boosts national income, enabling agents to increase current consumption. A decrease in interest rates raises consumption by discouraging savings and increasing disposable income.

Compared to the unfunded system, this study aimed to identify the optimal timing for transitioning to a funded system based on welfare outcomes. The most favorable scenario occurs when both shocks take place at time T=2, indicating that this timing leads to the highest welfare gains.

It is important to highlight that the models used in this thesis capture welfare changes only in response to the specific shocks applied within the Ecuadorian context, such as fluctuations in oil prices, given the country's dependence on this income source. However, the model does not account for potential changes in other factors, such as how pension system reforms might affect households' retirement decisions.

Moreover, the current model framework does not consider the effects of a pension reform on aggregate variables such as capital accumulation or commodity markets. While these aspects could be integrated into the OLG model, with relatively minor modifications, doing so would increase the computational complexity and cost of solving the model numerically. In addition, this study does not account for transitional dynamics between long-run equilibria. Analyzing such transitional shocks is important, as agents living through the transition may be worse off even if the reform benefits future generations. As supported by both this research and the broader literature, it cannot be assumed that the introduction of a fully funded pension system would be Pareto optimal.

From a political economy perspective, future research could explore whether it is possible to improve overall welfare by transitioning to a fully funded system, using a model calibrated more precisely to Ecuadorian data and potentially accounting for transitional dynamics and broader macroeconomic variables.

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Appendix

Households'Optimization Problem

The representative consumer maximizes their life-cycle utility, given by:

$$maxU_t = \frac{(C_t^y)^{1-\gamma} - 1}{1-\gamma} + \beta E_t \frac{(C_{t+1}^o)^{1-\gamma} - 1}{1-\gamma} \tag{A1}$$

subject to the following budget constraints:

$$C_t^y + a_{t+1} = w_t - \tau_t \tag{A2}$$

$$C_{t+1}^o = (1 + r_{t+1})a_{t+1} + T_{t+1} \tag{A3}$$

The consumer's problem is solved using the Lagrangian:

$$L = \frac{(C_t^y)^{1-\gamma} - 1}{1-\gamma} + \beta E_t \frac{(C_{t+1}^o)^{1-\gamma} - 1}{1-\gamma} + \lambda_1(w_t - \tau_t - C_t^y - a_{t+1})$$
$$+\lambda_2((1 + r_{t+1})a_{t+1} + T_{t+1} - C_{t+1}^o)$$

The first order conditions are:

$$\frac{\partial L}{\partial C_t^y} = (C_t^y)^{-\gamma} - \lambda_1 = 0 \tag{A4}$$

$$\frac{\partial L}{\partial C_{t+1}^o} = \beta(C_{t+1}^o)^{-\gamma} - \lambda_2 = 0 \tag{A5}$$

$$\frac{\partial L}{\partial a_{t+1}} = -\lambda_1 + \lambda_2(1 + r_{t+1}) = 0 \tag{A6}$$

Taking λ_1 and λ_2 from (A4) and (A5) we obtain:

$$\lambda_1 = (C_t^y)^{-\gamma} \tag{A7}$$

$$\lambda_2 = \beta(C_{t+1}^o)^{-\gamma} \tag{A8}$$

By combining (A7) and (A8) in in the first-order condition (A6), the intertemporal optimality condition is obtained:

$$(C_t^y)^{-\gamma} = E_t\beta(1 + r_{t+1})(C_{t+1}^o)^{-\gamma} \tag{A9}$$

Firm's Optimization Problem

The following linear production function problem defines the Firm's Optimization Problem:

$$Y_t = A_tL_t - w_tL_t \tag{A10}$$

The first-order condition for maximum is:

$$\frac{\partial Y_t}{\partial L_t} = A_t - w_t = 0 \tag{A11}$$

Here, the equilibrium is presented as:

$$A_t = w_t \tag{A12}$$

Social Security Transfers

The transfer mechanisms under both social security systems may yield different outcomes.

The Fully-Funded system represents transfers through the following equation:

$$T_{t+1} = (1 + r_{t+1})(\tau_t + \frac{QP^{oil}_{t+1} + IT}{N_t}) \quad \text{(A13)}$$

Using equations (A9) and A(13), the pension benefits under the Fully-Funded system are derived by substituting C^y_t (A2) and C^o_{t+1} (A3) in (A9):

$$(w_t - \tau_t - a_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})a_{t+1} + T_{t+1})^{-\gamma} \quad \text{(A14)}$$

By replacing into (A13), the equations (11),(17) and (A12) we obtain:

$$(A_t - \tau A_t - \hat{a}_{t+1}A_t)^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})\hat{a}_{t+1}A_t + (1 + r_{t+1})(\tau A_t + \frac{QP^{oil}_{t+1} + IT}{N_t}))^{-\gamma} \quad \text{(A15)}$$

By factoring out A_t and transferring it to the other side of the equation, we obtain:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})\hat{a}_{t+1} + (1 + r_{t+1})(\tau + \frac{QP^{oil}_{t+1} + IT}{N_tA_t}))^{-\gamma} \quad \text{(A16)}$$

By factoring out the common term $(1+r_{t+1})$ on the right-hand side and substituting equation (16), we obtain:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})^{1-\gamma}(\hat{a}_{t+1} + (\tau + Z_t))^{-\gamma} \quad \text{(A17)}$$

The transfer under both social security systems may yield different outcomes. The Pay-as-

You-Go system represents the transfers with the following equation:

$$T_{t+1} = \frac{N_{t+1}\tau_{t+1} + QP^{oil}_{t+1} + IT}{N_t} \tag{A18}$$

Using equations (A9) and A(18), the pension benefits under the Pay-as-You-Go system are derived by substituting C^y_t (A2) and C^o_{t+1} (A3) into equation (A9):

$$(w_t - \tau_t - a_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})a_{t+1} + T_{t+1})^{-\gamma} \tag{A19}$$

By replacing into (A12), the equations (11),(17) and (A18) we obtain:

$$(A_t - \tau A_t - \hat{a}_{t+1}A_t)^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})\hat{a}_{t+1}A_t + \frac{N_{t+1}\tau A_{t+1} + QP^{oil}_{t+1} + IT}{N_t})^{-\gamma} \tag{A20}$$

By factoring out A_t and isolating on the one side of the equation, we obtain:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})\hat{a}_{t+1} + \frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}\tau + \frac{QP^{oil}_{t+1} + IT}{N_tA_t})^{-\gamma} \tag{A21}$$

By multiplying and dividing the final term by $N_{t+1}A_{t+1}$ and factoring out the term $\frac{N_{t+1}A_{t+1}}{N_tAt}$, we obtain:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})((1 + r_{t+1})\hat{a}_{t+1} + \frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}(\tau + \frac{QP^{oil}_{t+1} + IT}{N_{t+1}A_{t+1}}))^{-\gamma} \tag{A22}$$

By replacing the population growth rate $\frac{N_{t+1}}{N_t} = (1 + \eta_{t+1})$, the technology growth rate

$\frac{A_{t+1}}{A_t} = (1 + g_{t+1})$ and equation (16), also, and by factoring out $(1 + r_{t+1})$, we obtain:

$$(1 - \tau - \hat{a}_{t+1})^{-\gamma} = E_t\beta(1 + r_{t+1})^{1-\gamma}\left(\hat{a}_{t+1} + \frac{(1 + \eta_{t+1})(1 + g_{t+1})}{1 + r_{t+1}}(\tau + Z_{t+1})\right)^{-\gamma} \tag{A23}$$

The transfer in equation (A13) can be re-expressed by substituting equation (11) and multiplying, and dividing by A_t in the last term:

$$T_{t+1} = (1 + r_{t+1})\left(\tau A_t + \frac{QP^{oil}_{t+1} + IT}{N_t A_t}A_t\right) \tag{A24}$$

By factoring out A_t and applying equation (16), we obtain:

$$T_{t+1} = (1 + r_{t+1})(\tau + Z_t)A_t \tag{A25}$$

The transfer in equation (A18) can be re-expressed by substituting equation (11), and by multiplying, and dividing the first term by A_t and the second term $N_{t+1}A_{t+1}A_t$. We then obtain:

$$T_{t+1} = \frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}\tau A_t + \frac{QP^{oil}_{t+1} + IT}{N_{t+1}A_{t+1}}\frac{N_{t+1}}{N_t}\frac{A_{t+1}}{A_t}A_t \tag{A26}$$

By substituting the population growth rate $\frac{N_{t+1}}{N_t} = (1 + \eta_{t+1})$, the technological growth rate $\frac{A_{t+1}}{A_t} = (1 + g_{t+1})$ and equation (16), and by factoring out the common terms $(1 + \eta_{t+1})$, $(1 + g_{t+1})$ and A_t, we obtain:

$$T_{t+1} = (1 + \eta_{t+1})(1 + g_{t+1})A_t(\tau + Z_{t+1}) \tag{A27}$$