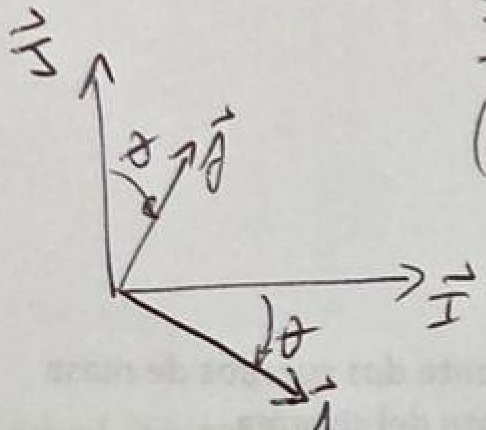
$\vec{I} = c\theta(\vec{i}) + s\theta(\vec{j})$

$(\vec{i}) = c\theta(\vec{I}) - s\theta(\vec{J})$



MCTG – 1051: MECÁNICA DE MAQUINARIA

EXAMEN MEJORAMIENTO

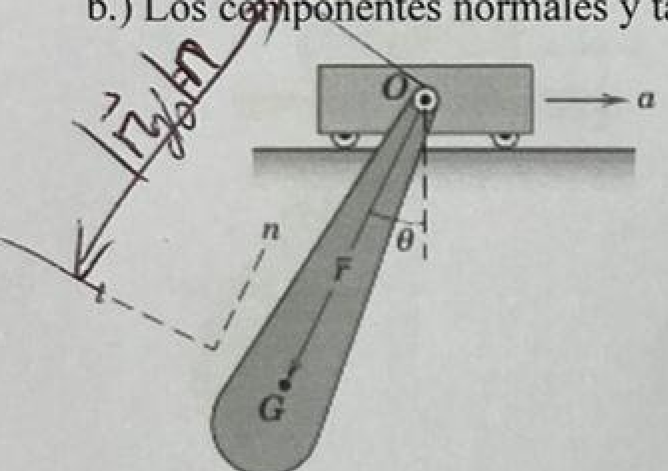
Livingston

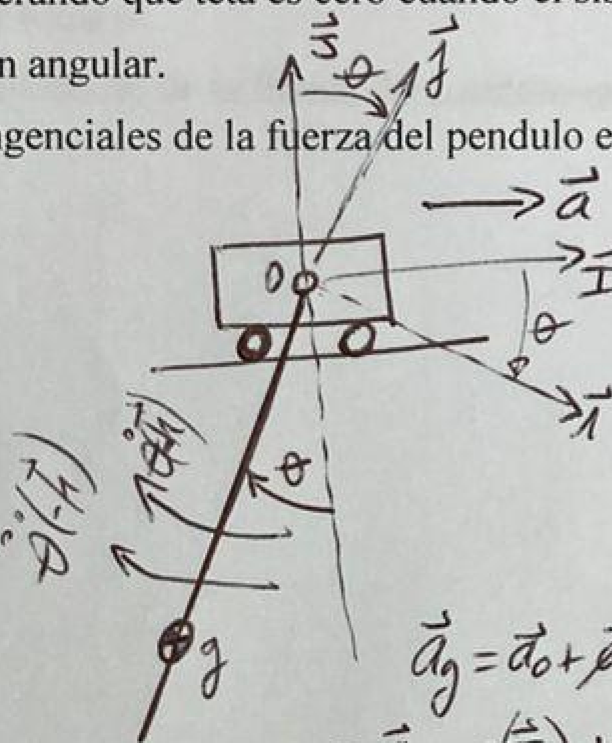
Tema 1 (60 puntos)

El pendulo compuesto de masa m, y un radio de giro ko, es libre de rotar respecto al punto O. La carretilla es sometida a una aceleracion constante a. Considerando que teta es cero cuando el sistema parte del reposo, determine:

a.) Una expresion para la aceleracion angular.

b.) Los componentes normales y tangenciales de la fuerza del pendulo en el punto O, en funcion de teta.





$\vec{V}_g = \vec{V}_o + \cancel{\vec{V}} + (\vec{\omega} \times \vec{r}_{g/o})$

$\vec{V}_g = V_o(\vec{I}) + (\dot{\theta}(+\vec{k}) \times r(+\vec{j}))$

$\boxed{\vec{V}_g = V_o(\vec{I}) - (r\dot{\theta})(\vec{i})}$

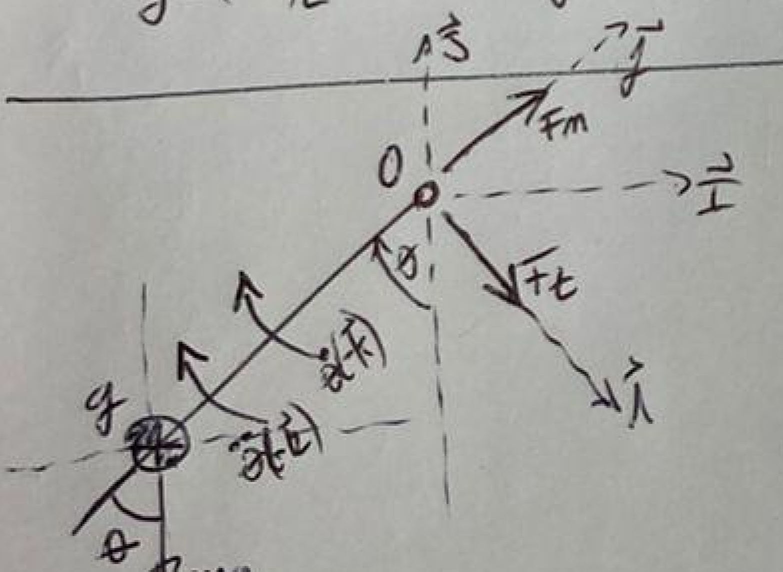
$\vec{a}_g = \vec{a}_o + \cancel{\vec{a}} + 2(\vec{\omega} \times \cancel{\vec{v}}) + (\vec{\alpha} \times \vec{r}_{g/o}) + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{g/o})$

$\rightarrow \vec{a}_g = a(\vec{I}) + \underbrace{(\ddot{\theta}(+\vec{k}) \times r(+\vec{j}))}_{-\vec{i}} + \Big[\dot{\theta}(+\vec{k}) \times \underbrace{(\dot{\theta}(+\vec{k}) \times r(+\vec{j}))}_{+\vec{i}}\Big]$

$\Rightarrow \boxed{\vec{a}_g = a(\vec{I}) - (r\ddot{\theta})(\vec{i}) + r(\dot{\theta})^2(\vec{j})}$

$\vec{I} = c\theta(\vec{i}) + s\theta(\vec{j})$

$\hookrightarrow \vec{a}_g = (a)\left[c\theta(\vec{i}) + s\theta(\vec{j})\right] - (r\ddot{\theta})(\vec{i}) + r(\dot{\theta})^2(\vec{j}) = \boxed{(\vec{i})\left[a(c\theta) - r(\ddot{\theta})\right] + (\vec{j})\left[a(s\theta) + r(\dot{\theta})^2\right] = \vec{a}_g}$



$\nearrow \sum F_n = m(a_g(\vec{j}))$

$+F_n - mg(c\theta) = (m)\left[a(s\theta) + r(\dot{\theta})^2\right]$

$\hookrightarrow \boxed{F_n = (m)\left[a(s\theta) + r(\dot{\theta})^2\right] + mg(c\theta)}$

Parte b

$(\vec{i}) \sum F_t = m a_g(\vec{i})$

$+F_t + mg\,s\theta = m\left[a(c\theta) - r\ddot{\theta}\right]$

$\hookrightarrow \boxed{F_t = m\left[a(c\theta) - r\ddot{\theta}\right] - mg\,s\theta}$

$\sum \vec{M}_o = \vec{I_o \alpha} \rightarrow \circlearrowleft \sum \vec{M}_o = I_o(\ddot{\theta})(-\vec{k})$

$\rightarrow + mg(r s\theta)(\vec{k}) = (I_o)(\ddot{\theta})(-\vec{k})$

$\rightarrow \cancel{m}g r s\theta = -\cancel{m}(k_o)^2\ddot{\theta}$

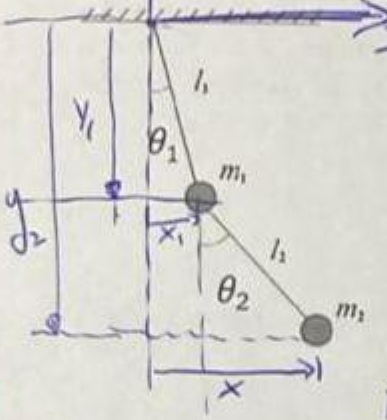
$\rightarrow \boxed{\ddot{\theta} = \dfrac{g r s\theta}{(k_o)^2}}$

$\hookrightarrow$ Pregunta

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = \cancel{Q_i}} \quad Q_i = 0$$

Tema 2 ([redacted]tos)

Dos masas puntuales (m1 y m2) forman un doble péndulo. Las masas están unidas mediante dos cuerpos de masa despreciable y longitud l1 y l2, respectivamente. Determinar las ecuaciones de movimiento del sistema.



Posición:

$$m_1\begin{cases} x_1 = l_1 \sin\theta_1 \\ y_1 = -l_1\cos\theta_1 \end{cases}$$

$$m_2\begin{cases} x_2 = l_1\sin\theta_1 + l_2\sin\theta_2 \\ y_2 = -l_1\cos\theta_1 - l_2\cos\theta_2 \end{cases}$$

Velocidad:

$$\dot x_1 = l_1\dot\theta_1\cos\theta_1$$
$$\dot y_1 = +l_1\dot\theta_1\sin\theta_1$$
$$\dot x_2 = l_1\dot\theta_1\cos\theta_1 + l_2\dot\theta_2\cos\theta_2$$
$$\dot y_2 = +l_1\dot\theta_1\sin\theta_1 + l_2\dot\theta_2\sin\theta_2$$

Energía Cinética: $T = T_1 + T_2 = \frac{1}{2}m_1(\dot x_1^2 + \dot y_1^2) + \frac{1}{2}m_2(\dot x_2^2 + \dot y_2^2)$

$$\dot x_1^2 = (l_1\dot\theta_1)^2\cos^2\theta_1 \ ; \ \dot y_1^2 = (l_1\dot\theta_1)^2\sin^2\theta_1 \Rightarrow (\dot x_1^2 + \dot y_1^2) = (l_1\dot\theta_1)^2$$

$$(\dot x_2)^2 = (l_1\dot\theta_1)^2\cos^2\theta_1 + 2(l_1\dot\theta_1)(l_2\dot\theta_2)\cos\theta_1\cos\theta_2 + (l_2\dot\theta_2)^2\cos^2\theta_2$$

$$(\dot y_2)^2 = (l_1\dot\theta_1)^2\sin^2\theta_1 + 2(l_1\dot\theta_1)(l_2\dot\theta_2)\sin\theta_1\sin\theta_2 + (l_2\dot\theta_2)^2\sin^2\theta_2$$

$$(\dot x_2)^2 + (\dot y_2)^2 = (l_1\dot\theta_1)^2 + (l_2\dot\theta_2)^2 + 2(l_1l_2)(\dot\theta_1\dot\theta_2)\cos(\theta_1-\theta_2)$$

$$\Rightarrow \boxed{T = \frac{1}{2}m_1(l_1\dot\theta_1)^2 + \frac{1}{2}m_2\left[(l_1\dot\theta_1)^2 + (l_2\dot\theta_2)^2 + 2(l_1l_2)(\dot\theta_1\dot\theta_2)\cos(\theta_1-\theta_2)\right]}$$

Energía Potencial: $V = mgh \rightarrow V_1 = m_1 g y_1 \ ; \ V_2 = m_2 g y_2$

$$\rightarrow \boxed{V = -(m_1 g l_1\cos\theta_1) - (m_2 g)(l_1\cos\theta_1 + l_2\cos\theta_2)}$$

$$\boxed{L = T - V}$$
Lagrangiano

$$\frac{\partial L}{\partial \dot q_i} \rightarrow \boxed{q_1 = \theta_1 \wedge q_2 = \theta_2}$$

$$q_1 = \theta_1 \Rightarrow \frac{\partial L}{\partial\dot\theta_1} = m_1(l_1)^2\dot\theta_1 + m_2(l_1)^2(\dot\theta_1) + 2(l_1l_2)(\dot\theta_2)\cos(\theta_1-\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta_1}\right) = m_1(l_1)^2\ddot\theta_1 + m_2(l_1)^2(\ddot\theta_1) + m_2(l_1l_2)(\ddot\theta_2)\cos(\theta_1-\theta_2) - m_2(l_1l_2)(\dot\theta_2)(\dot\theta_1 - \dot\theta_2)\sin(\theta_1-\theta_2)$$

$$\frac{\partial V}{\partial\theta_1} = m_1 g l_1 s\theta_1 + m_2 g l_1 s\theta_1 = \boxed{(m_1+m_2)l_1 s\theta_1}$$

$$\frac{\partial T}{\partial\theta_1} = -m_2(l_1l_2)(\dot\theta_1\dot\theta_2)\sin(\theta_1-\theta_2)$$

$$\boxed{\begin{aligned}(m_1l_1^2 + m_2l_1^2)(\ddot\theta_1) + m_2(l_1l_2)(\ddot\theta_2)\cos(\theta_1-\theta_2) - m_2(l_1l_2)(\dot\theta_2)(\dot\theta_1-\dot\theta_2)\sin(\theta_1-\theta_2) \\ + m_2(l_1l_2)(\dot\theta_1\dot\theta_2)\sin(\theta_1-\theta_2) - (m_1+m_2)l_1 s\theta_1 = 0\end{aligned}}$$

$$q_2 = \theta_2 \Rightarrow \frac{\partial L}{\partial\dot\theta_2} = +m_2(l_2)^2(\dot\theta_2) + (m_2)(l_1l_2)(\dot\theta_1)\cos(\theta_1-\theta_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta_2}\right) = m_2(l_2)^2(\ddot\theta_2) + (m_2)(l_1l_2)(\ddot\theta_1)\cos(\theta_1-\theta_2) - m_2(l_1l_2)(\dot\theta_1)(\dot\theta_1-\dot\theta_2)\sin(\theta_1-\theta_2)$$

$$\frac{\partial L}{\partial\theta_2} = m_2(l_1l_2)(\dot\theta_1\dot\theta_2)\sin(\theta_1-\theta_2) - m_2 g l_2\sin\theta_2$$

$$\Rightarrow \boxed{\begin{aligned}m_2(l_2)^2(\ddot\theta_2) + m_2(l_1l_2)(\ddot\theta_1)\cos(\theta_1-\theta_2) - m_2(l_1l_2)(\dot\theta_1)(\dot\theta_1-\dot\theta_2)\sin(\theta_1-\theta_2) - m_2(l_1l_2)(\dot\theta_1\dot\theta_2)\sin(\theta_1-\theta_2) \\ + m_2 g l_2\sin\theta_2 = 0\end{aligned}}$$