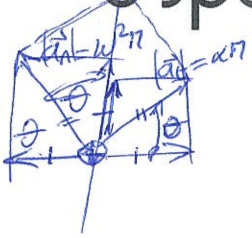
$a_n = (10\ rad/s)^2(0.5) = \boxed{50\ m/s^2}$

$a_t = (2\ rad/s^2)(0.5) = \boxed{1\ m/s^2}$

→ Solución 2025-1T

MCTG – 1051: MECÁNICA DE MAQUINARIA

EXAMEN FINAL – 2025 – 1T



$\boxed{a_{2x} = -(a_n)(S\theta) + (a_t)(C\theta)}$

$\boxed{a_{2y} = +(a_n)(C\theta) + (a_t)(S\theta)}$ →

$a_{2x} = -(50)\sin 30° + (1)(\cos 30°) = \boxed{-24.134\ (m/s^2)}$

$a_{2y} = +(50)\cos 30° + (1)(\sin 30°) = \boxed{43.801\ (m/s^2)}$

centro de Masa

Tema 1 (40puntos)

El sistema mostrado en la figura opera con una fuerza constante de P=100 **j** N, para lo cual se debe considerar las siguientes condiciones del sistema:

I.) r_2= 1m, φ_2=30°, ω_2= 10 (**k**) rad/s, α_2=2 (**k**) rad/s^2. Los centros de masa de los eslabones 2 y 3, están en sus respectivos centros geométricos. Además, considerar las masas de m_2=5 Kg, m_3= 5Kg, m_4= 15 Kg, y las inercias de I_{G2}=0.02 N·m·s^2, I_{G3}=0.12 N·m·s^2, I_{G4}=0.08 N·m·s^2. La gravedad está actuando perpendicular al plano y se puede despreciar la fricción.

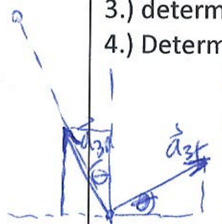
Requerimientos:

1.) Dibuje los diagramas de cuerpo libre ✓

2.) Escribir las ecuciones que describen los movimientos de los eslabones ✓

3.) determine las magnitudes y direcciones de las fuerzas en las juntas ✓

4.) Determine la magnitud y la direccion del torque T2 ✓



$a_{3n} = (10\ rad/s)^2(1) = \boxed{100\ m/s^2}$

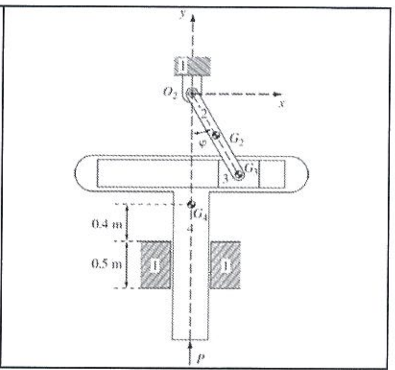
$a_{3t} = (2\ rad/s^2)(1) = \boxed{2\ m/s^2}$

$a_{3x} = -(a_{3n})(\sin\theta) + (a_{3t})\cos\theta$

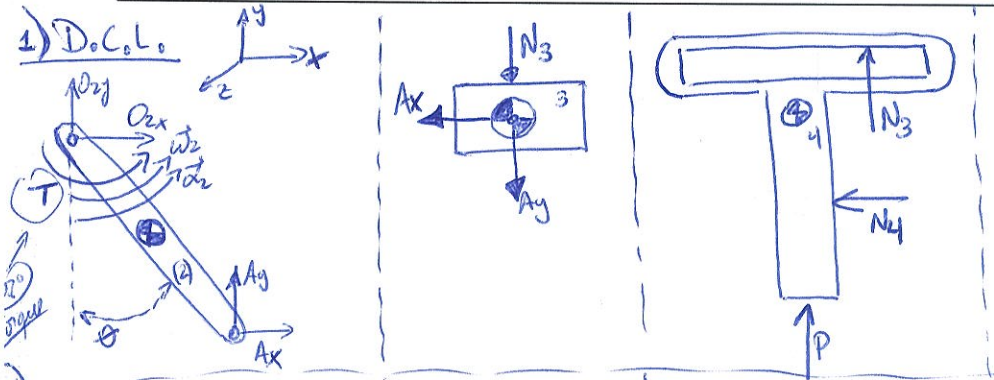
$a_{3y} = (a_{3n})(\cos\theta) + (a_{3t})(\sin\theta)$

$a_{3x} = -(100 \times \sin 30°) + (2 \times \cos 30°) = -48.268\ (m/s^2)$

$a_{3y} = (100 \times \cos 30°) + (2 \times \sin 30°) = +87.603\ (m/s^2)$



1) D.C.L.



cuerpo #2

$\Sigma F_x = m_2 a_{2x}$

$\boxed{+O_{2x} + A_x = m_2 a_{2x}}$

$\Sigma F_y = m_2 a_{2y}$

$\boxed{+O_{2y} + A_y = m_2 a_{2y}}$

cuerpo #3

$\Sigma F_x = m_3 a_{3x}$

$\boxed{-A_x = m_3 a_{3x}}$ ✓

$\Sigma F_y = m_3 a_{3y}$

$\boxed{-N_3 - A_y = m_3 a_{3y}}$ ✓

Cuerpo #4

$\Sigma F_x = m_4 a_{4x}^{\,0} \Rightarrow \boxed{N_4 = 0}$

$\Sigma F_y = m_4 a_{4y}$

$\boxed{+N_3 + P = m_4 a_{4y}}$ ✓

$a_{g2x} = -24.13\ (m/s^2)$ | $a_{3x} = -48.27\ (m/s^2)$

$a_{g2y} = 43.80\ (m/s^2)$ | $a_{3y} = +87.60\ (m/s^2)$

$\boxed{a_{g4} = a_{3y}} = 87.60\ (m/s^2)$

$a_{4y} =$

$\boxed{a_{4x} = 0}$

$\Sigma \vec{M}_{O_2} = (I_{g2}\vec{\alpha}_2) + (\vec{S}_{g2/O_2} \times m_2 \vec{a}_{g2})$

$\boxed{T + A_y(1)\sin 30° + A_x(1)\cos 30° = (I_{g2}\alpha_2) - (0.5)(\cos 30°)(a_{2x}) + (0.5)(\sin 30°)(a_{2y})}$

3) $N_3 = (m_4 a_{4y}) - P = (15\ kg)(87.60\ m/s^2) - 100 = \boxed{1214\ (N)} = N_3$ → $\boxed{\vec{N}_3 = N_3(\vec{j})}$

$A_y = -N_3 - m_3 a_{3y} = -(1214) - (5\ kg)(87.60) \Rightarrow \boxed{A_y = -1652\ (N)}$ → $\boxed{\vec{A} = +241.35(\vec{i}) - 1652(\vec{j})}$

$A_x = -m_3 a_{3x} = -(5\ kg)(-48.27) = \boxed{241.35\ (N)} = A_x$

$T = (I_{g2}\alpha_2) - (0.5\cos 30°)(a_{2x}) + (0.5)(\sin 30°)(a_{2y}) - A_y(1\sin 30°) - A_x(1)\cos 30°$

$T = (0.02)(2) - (0.5\cos 30°)(24.13) + (0.5\sin 30°)(43.80) - (-1652)(\sin 30°) - (241.35)(\cos 30°) \Rightarrow \boxed{T_2 = 617.53\ [Nm]}$

$\boxed{\vec{T}_2 = (T_2)(\vec{k})}$

$O_{2y} = (m_2 a_{2y}) - A_y = (5\ kg)(43.80) - (-1652) = 1871$

$O_{2x} = (m_2 a_{2x}) - A_x = (5\ kg)(-24.13) - (241.35) = -362$

$\boxed{\vec{O}_2 = -362(\vec{i}) + 1871(\vec{j})}$

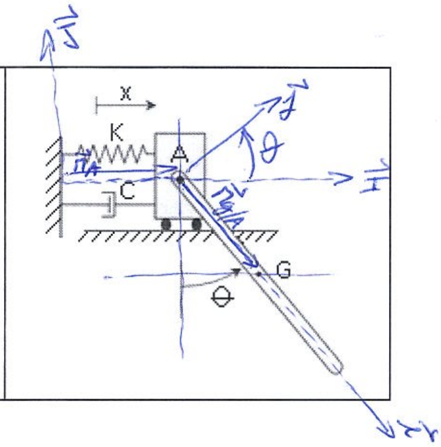
Solución 2023-1T

MCTG – 1051: MECÁNICA DE MAQUINARIA

espol

Tema 2 (60puntos)

El cuerpo A (de masa m) se desliza sobre una superficie horizontal sin fricción, junto con una barra fina y homogénea (longitud L y masa M), que está unida al cuerpo A, mediante un pasador (sin fricción), como se muestra en la figura. Un resorte y un amortiguador conectan al cuerpo A con la bancada. La coordenada x describe el movimiento de A (positivo hacia la derecha, y el resorte está sin estiramiento cuando x=0). La coordenada θ describe la orientación de la barra, con θ positivo en sentido anti horario, desde el eje vertical. Determinar las ecuaciones de movimiento del sistema, usando las coordenadas x y θ.



I.) Sistema de referencia

II.) Posición/Velocidad/Aceleración

$\vec{r}_A = x(\vec{I})$; $\vec{r}_g = \vec{r}_A + \vec{r}_{g/A}$

$\dot{\vec{r}}_A = \dot{x}(\vec{I})$; $\boxed{\vec{r}_g = x(\vec{I}) + \left(\frac{l}{2}\right)(\vec{i})}$

$\ddot{\vec{r}}_A = \ddot{x}(\vec{I})$

$\Rightarrow \dot{\vec{r}}_g = \dot{x}(\vec{I}) + \left(\frac{l}{2}\right)(\dot{\vec{i}})$; $\begin{cases} \vec{\omega} = \dot{\theta}(\vec{k}) \\ \dot{\vec{i}} = (\vec{\omega}\times\vec{i}) = (\dot{\theta}\vec{k}\times\vec{i}) = \dot{\theta}(\vec{j}) \\ \vec{j} = \cos\theta(\vec{I}) + \sin\theta(\vec{J}) \end{cases}$

$\Rightarrow \dot{\vec{r}}_g = \dot{x}(\vec{I}) + \left(\frac{l}{2}\right)\dot{\theta}\left[\cos\theta(\vec{I}) + \sin\theta(\vec{J})\right] = \boxed{\underbrace{(\vec{I})\left[\dot{x} + \left(\frac{l}{2}\right)(\dot{\theta})\cos\theta\right] + (\vec{J})\left[\left(\frac{l}{2}\right)(\dot{\theta})\sin\theta\right]}_{=\dot{\vec{r}}_g}}$

III.) Energía

$T_A = \frac{1}{2}m_A(\dot{\vec{r}}_A\cdot\dot{\vec{r}}_A) = \boxed{\frac{1}{2}m(\dot{x})^2}$

$T_B = \frac{1}{2}m_B(\dot{\vec{r}}_g\cdot\dot{\vec{r}}_g) + \frac{1}{2}I_B\omega^2 = \frac{1}{2}(M)\left[(\vec{I})\left[\dot{x} + \left(\frac{l}{2}\right)(\dot{\theta})\cos\theta\right] + (\vec{J})\left[\left(\frac{l}{2}\right)(\dot{\theta})\sin\theta\right]\right]\cdot\left[(\vec{I})\left[\dot{x} + \left(\frac{l}{2}\right)(\dot{\theta})\cos\theta\right] + (\vec{J})\left(\frac{l}{2}\dot{\theta}\sin\theta\right)\right] + \frac{1}{2}I_B(\dot{\theta})^2$

$T_B = \frac{1}{2}(M)\left[\left(\dot{x} + \frac{l}{2}\dot{\theta}\cos\theta\right)^2 + \left(\frac{l}{2}\dot{\theta}\sin\theta\right)^2\right] + \frac{1}{2}I_B(\dot{\theta})^2 = \frac{1}{2}(M)\left[(\dot{x})^2 + (\dot{x})(l)(\dot{\theta})\cos\theta + \left(\frac{l}{2}\right)^2(\dot{\theta})^2\cos^2\theta + \left(\frac{l}{2}\right)^2(\dot{\theta})^2\sin^2\theta\right] + \frac{1}{2}I_B(\dot{\theta})^2$

$\Rightarrow \boxed{T_B = \frac{1}{2}(M)\left[(\dot{x})^2 + (\dot{x})(l)(\dot{\theta})\cos\theta + \left(\frac{l}{2}\right)^2(\dot{\theta})^2\right] + \frac{1}{2}I_B(\dot{\theta})^2}$

$\hookrightarrow \boxed{T_B = \frac{1}{2}(M)(\dot{x})^2 + M\left(\frac{l}{2}\right)(\dot{x})(\dot{\theta})\cos\theta + \left[\frac{1}{2}M\left(\frac{l}{2}\right)^2 + \frac{1}{2}I_B\right](\dot{\theta})^2}$

$\boxed{V_g = -m_B g\left(\frac{l}{2}\right)\cos\theta}$; $\boxed{V_k = \frac{1}{2}k(x)^2}$; $\boxed{R = \frac{1}{2}c(\dot{x})^2}$

IV.) Lagrange : $\boxed{\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i}$; $L = T - V$; $V = V_g + V_k$

$\boxed{q_1 = x} \Rightarrow \frac{\partial T}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}m(\dot{x})^2 + \frac{1}{2}(M)(\dot{x})^2 + (M)\left(\frac{l}{2}\right)(\dot{x})(\dot{\theta})\cos\theta + \left[\frac{1}{2}M\left(\frac{l}{2}\right)^2 + \frac{1}{2}I_B\right](\dot{\theta})^2\right) = m(\dot{x}) + M(\dot{x}) + M\left(\frac{l}{2}\right)\dot{\theta}\cos\theta$

$\Rightarrow \boxed{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m\ddot{x} + M\ddot{x} + M\left(\frac{l}{2}\right)\ddot{\theta}\cos\theta - M\left(\frac{l}{2}\right)(\dot{\theta})^2\sin\theta}$; $\boxed{\frac{\partial V}{\partial \dot{x}} = 0}$; $\frac{\partial R}{\partial \dot{x}} = c(\dot{x})$; $Q_1 = 0$

$\Rightarrow$ $\frac{\partial T}{\partial x} = 0$; $\frac{\partial V_g}{\partial x} = 0$; $\frac{\partial V_k}{\partial x} = kx$

$\Rightarrow \boxed{m\ddot{x} + M\ddot{x} + M\left(\frac{l}{2}\right)\ddot{\theta}\cos\theta - M\left(\frac{l}{2}\right)(\dot{\theta})^2\sin\theta + (kx) + c\dot{x} = 0}$ $\rightsquigarrow$ Ecuación de movimiento #1

$\hookrightarrow \boxed{\ddot{x}(M+m) + (\ddot{\theta})\left(M\left(\frac{l}{2}\right)\cos\theta\right) - M\frac{l}{2}(\dot{\theta})^2\sin\theta + c(\dot{x}) + kx = 0}$

$q_2 = \theta$

$\frac{\partial T}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}}\left(\frac{1}{2}m(\dot{x})^2 + \frac{1}{2}M(\dot{x})^2 + M\left(\frac{l}{2}\right)\dot{x}\dot{\theta}\cos\theta + \left[\frac{1}{2}M\left(\frac{l}{2}\right)^2 + \frac{1}{2}I_B\right](\dot{\theta})^2\right) = M\left(\frac{l}{2}\right)\dot{x}\cos\theta + \left[M\left(\frac{l}{2}\right)^2 + I_B\right](\dot{\theta})$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \boxed{M\left(\frac{l}{2}\right)\ddot{x}\cos\theta - M\left(\frac{l}{2}\right)\dot{x}\dot{\theta}\sin\theta + \left[M\left(\frac{l}{2}\right)^2 + I_B\right](\ddot{\theta})}$; $\frac{\partial V}{\partial \dot{\theta}} = 0$; $\frac{\partial R}{\partial \dot{\theta}} = 0$; $Q_2 = 0$; $\frac{\partial T}{\partial \theta} = -M\frac{l}{2}\dot{x}(\dot{\theta})\sin\theta$

$\frac{\partial V_g}{\partial \theta} = +Mg\left(\frac{l}{2}\right)\sin\theta$;

$\Rightarrow \boxed{M\left(\frac{l}{2}\right)\ddot{x}\cos\theta - M\left(\frac{l}{2}\right)\dot{x}\dot{\theta}\sin\theta + \left[M\left(\frac{l}{2}\right)^2 + I_B\right](\ddot{\theta}) + M\left(\frac{l}{2}\right)(\dot{x})(\dot{\theta})\sin\theta - Mg\left(\frac{l}{2}\right)\sin\theta = 0}$ $\rightsquigarrow$ Ecuación de Movimiento #2